TE TAI ŌHANGA
THE TREASURY

Shocks and Scenarios Analysis Using a Stochastic Neoclassical Growth Model

Background Paper for the

2021 Statement on the Long-term Fiscal Position

September 2021

Te Kāwanatanga o Aotearoa
New Zealand Government

BACKGROUND PAPER FOR THE 2021 STATEMENT ON THE LONG-TERM FISCAL POSITION:	Shocks and Scenarios Analysis Using a Stochastic Neoclassical Growth Model
MONTH/YEAR	September 2021
AUTHOR	Andrew Binning New Zealand Treasury No. 1 The Terrace Wellington New Zealand Email: andrew.binning@treasury.govt.nz Telephone: ++64 +4 917 6010
URL	Treasury website at September 2021: https://www.treasury.govt.nz/publications/background/ltfs-21-shocks-scenarios-analysis-using-stochastic-neoclassical-growth-model
ACKNOWLEDGEMENTS	I thank Jean-Pierre Andre, Matthew Appleby, Matthew Bell, Steve Cantwell, Karsten Chipeniuk, Shane Domican, Craig Fookes, Margaret Galt, Peter Gardiner, John Janssen, Anita King, Sam King, Natalie Labuschagne, Udayan Mukherjee, Murat Özbilgin, Oscar Parkyn, Hemant Passi, Bettina Schaer, Nathan Spence and Niven Winchester for their help and their useful comments. All mistakes and errors are my own.
NZ TREASURY	New Zealand Treasury PO Box 3724 Wellington 6008 NEW ZEALAND Email: information@treasury.govt.nz Telephone: +64 4 472 2733 Website: www.treasury.govt.nz

Abstract

The Public Finance Act 1989 requires the New Zealand Treasury to produce a long-term fiscal statement at least once every four years. With COVID delaying the publication of the 2020 long-term fiscal statement it was decided to combine the 2021 long-term fiscal statement with Treasury's inaugural long-term insights briefing. While the long-term fiscal statement presents how the Crown's fiscal position could evolve over the next 40 years and some of the fiscal challenges they could face, the long-term insights briefing, a new requirement under the Public Services Act 2020, includes analysis and policy options that current and future governments could consider when addressing these future fiscal challenges. The ageing population, increasing health costs and climate change present challenges that could have large fiscal consequences for both current and future generations. While the implications of an ageing population and increasing health costs have been covered extensively in previous long-term fiscal statements, the fiscal implications of climate change are being considered for the first time. As part of the supporting analysis for the long-term fiscal statement I produce a number of scenarios illustrating the fiscal implications of an ageing population, increasing health costs and some specific aspects of climate change. I employ a stochastic neoclassical growth model, for the first time, to produce the shocks and scenarios analysis. Treating the ageing population and climate change separately, I find that net debt can be kept close to 48 percent of GDP by 2061 in the baseline ageing population scenario, but average tax rates on capital and labour income and consumption expenditure need to be raised by 10 percentage points, 6.4 percentage points and 5 percentage points respectively. The projections are re-run under a number of alternative assumptions, including subjecting the model economy to economic and physical shocks, to see how the results change. I also investigate the fiscal implications of extreme weather events, which are predicted to increase in frequency and intensity as the effects of climate change become more pronounced.

JEL CLASSIFICATION E13
E17
E62
H51
H55
H63

KEYWORDS Ageing Population, Increasing Health Costs, Climate Change, Shocks and Scenarios Analysis, Stochastic Neoclassical Growth Model

Executive Summary

At least once every four years the New Zealand Treasury is required, under the Public Finance Act 1989, to produce a long-term fiscal statement, outlining how the Crown's fiscal position could evolve over the next 40 years. With the onset of COVID the publication of the 2020 long-term fiscal statement was delayed. The updated 2021 long-term fiscal statement has been combined with the Treasury's inaugural long-term insights briefing. The long-term insights briefing, a new requirement under the Public Service Act 2020, outlines future fiscal challenges and provides analysis and policy options government could consider when responding to these challenges. The largest fiscal pressures, likely to affect revenue, expenditure and net government debt, are the ageing population, increasing health expenditure and climate change. While the fiscal impacts of an ageing population and increasing health expenditure have been extensively covered in previous long-term fiscal statements, this is the first time the fiscal implications of climate change are considered. This is also the first time shocks and scenarios analysis is produced using a general equilibrium model (in this case, a stochastic neoclassical growth model), to support the long-term fiscal statement, which I document in this paper.

Analysis from the long-term fiscal model (LTFM), a spreadsheet-style accounting model, has formed the basis of the modelling work for previous long-term fiscal statements. The LTFM is an extremely useful tool, providing detailed projections of government expenditure, revenue and net government debt, under the assumption that policy does not respond and there is no feedback to the rest of the economy. However, the LTFM (like all models) has its limitations. In particular, it does not capture behavioural or feedback responses, and it is deterministic, which rules out the investigation of uncertainty. I address these issues in this paper by abandoning spreadsheet-style accounting models in favour of a general equilibrium model. More specifically I develop a stochastic neoclassical growth model (NCGM), with a government sector, to carry out analysis investigating the fiscal impacts of an ageing population, increased health spending and some aspects related to climate change. The model I develop is plain vanilla in many regards, sharing a number of features with the textbook growth model. Government plays a key role stabilising net debt around its long-run target. The NCGM provides complementary analysis to the LTFM.

In the baseline ageing population scenario key spending tracks from the NCGM are matched with their counterparts in the LTFM and net debt as a share of GDP is kept reasonably stable around its target over the 40 year reporting period. However, this requires substantial increases in tax rates, with the average tax rate on labour increasing by 6.4 percentage points, the average tax rate on capital income increasing by 10 percentage points, and the average tax rate on consumption expenditure increasing by 5 percentage points, resulting in a GDP path that is 3.7 percent lower than what have prevailed in the absence of these spending pressures in 2061.

I produce a range of alternative scenarios where some of these assumptions are altered or the model economy is subjected to economic and physical shocks. The main results from these scenarios can be summarised as follows:

- The recessions modelled raise net debt to GDP by between 11 and 13 percentage points, and require reasonably aggressive tax responses from government to return net debt to target before the next recession.
- A large earthquake raises net debt to GDP by 12 percentage points, similar to the recessions scenario. The overall fall in GDP is smaller compared with the recessions scenario, due to the faster decline and recovery in trend total factor productivity and the rebuild which factors in a healthy degree of "building back better".

- Fast fiscal consolidations are more costly (in a GDP sense) in the short run, but generate more benefit in the long run as debt servicing costs are reduced. Slow fiscal consolidations spread the economic costs over a longer period of time, resulting in smaller costs per period. However, elevated debt in the slow consolidation scenario translates into higher tax rates and lower spending over a longer period of time and a larger cumulative GDP cost.

- Delayed fiscal responses to increasing debt generate benefits in the short run, as additional government spending increases aggregate demand without the costs of higher taxation. However, when government decides to stabilise debt at a higher level, or consolidate, taxes need to be raised rapidly to fund higher debt servicing costs and the higher primary surpluses required to bring debt back to target. These higher taxes reduce economic activity and the level of GDP. In most scenarios investigated the net gains from delaying are outweighed by the costs of stabilising or consolidating debt.

I produce separate stochastic scenarios that investigate the fiscal and economic implications of changes in the frequency of droughts and the intensity of storms due to the effects of climate change. In the droughts scenario, the increase in drought frequency could lower GDP by 0.5 percent relative to trend, on average, by 2061, while net debt to GDP could be 1.2 percentage points higher on average. In the storms scenario, the increase in storm intensity suggests that GDP could be 0.7 percent lower than trend, on average, by 2061 and net debt to GDP could be 3 percentage points higher, on average. I note that droughts and storms add a measurable amount of volatility to both the net debt and GDP projections.

Contents

List of Tables

List of Figures

Shocks and Scenarios Analysis Using a Stochastic Neoclassical Growth Model

1. Introduction

At least once every four years the New Zealand Treasury is required, under the Public Finance Act 1989, to produce a statement on the Crown's long-term fiscal position. With the onset of COVID the publication of the 2020 long-term fiscal statement was delayed. The updated 2021 publication has now been combined with the inaugural long-term insights briefing. While the long-term fiscal statement outlines how the Crown's fiscal position could evolve over the next 40 years, illustrating some of the likely fiscal challenges and trade-offs faced by government, the long-term insights briefing, which is a new requirement under the Public Service Act 2020, covers some of the policy options governments could consider when responding to these challenges. Some of the largest revenue and expenditure pressures are expected to be the ageing population, increasing health costs and climate change. While the fiscal consequences of the ageing population and increasing health expenditure have been extensively covered in previous long-term fiscal statements and supporting papers, the fiscal consequences of climate change are being included in the long-term fiscal statement for the first time. As part of the analytical work supporting the combined long-term fiscal statement and insights briefing, I present shocks and scenarios analysis that highlights some of the potential revenue and expenditure pressures associated with the ageing population and increased health expenditure, and some specific aspects related to climate change. These scenarios are produced using Treasury's new stochastic neoclassical growth model (NCGM), which I document in this paper. This is the first time Treasury has used a general equilibrium model to produce projections and scenarios analysis supporting the long-term fiscal statement. The NCGM is able to capture some of the behavioural and feedback mechanisms missing from the modelling work in previous long-term fiscal statements. It can also be used to investigate uncertainty in a coherent manner. I produce a number of ageing population and climate change scenarios using the model for a range of shocks and assumptions, illustrating potential fiscal pressures and trade-offs faced by government.

Previous long-term fiscal statements have been built around the analysis from the long-term fiscal model (LTFM) (see Bell and Piscetek, 2016, for example). The LTFM is a spreadsheet accounting model that combines a number of accounting identities with exogenous projections for interest rates, labour productivity, inflation and a set of demographic factors. It is an extremely useful tool for calculating government spending, transfers spending and government debt paths under the assumption that policy does not change, and that there is no feedback or behavioural response from economic agents.[1] Bell (2021) documents the current iteration of the LTFM. Debt projections from this model, under current policy settings, usually explode, illustrating the unsustainability of current policy.[2] The conclusion drawn from these results is that policy needs to change to ensure a sustainable debt path. The Office of the Auditor General (see OAG, 2017) has noted a lack of feedback effects, analysis of uncertainty, and sensitivity testing in their assessment of the modelling work supporting previous long-term fiscal statements. Similar observations have been made by Ter-Minassian (2014) and Buckle (2018).

[1] Throughout the paper, agents refers to households (consumers), firms (producers) and government.

[2] In Bell and Piscetek (2016) net debt to GDP gets to 60 percent by 2040 and 200 percent by 2060. In Bell (2021) net debt to GDP gets to 84 percent in 2045 and 197 percent by 2061.

The Congressional Budget Office (CBO) produces similar types of projections for the US. Using a spreadsheet accounting model, they project government debt forward under current policy settings.[3] They too find government debt explodes over the 50 year projection horizon, because policy is not allowed to respond to rising debt.[4] Eric Leeper in a series of papers (Leeper, 2010; Leeper, 2011) has raised issues with the assumptions used to construct the CBO's projections. In particular he notes that the analysis does not come from an economic model, that it forms a baseline the public know will not happen and that it does not provide any information about how policy could or should respond. Investors, who have full knowledge and understanding of the CBO's projections (which are public information), continue buying government bonds without demanding a risk premium. This is an indication that expectations about future debt dynamics are at odds with the assumptions used to construct the CBO's projections. Leeper points out that debt projections where debt gets to excessively high levels are problematic because they entail higher debt servicing costs, forcing government to either cut expenditure or raise tax rates. Stabilising debt at higher levels requires higher primary surpluses. Politics and the Laffer curve impose limits on the maximum amount of tax revenue that can be collected. At the same time it is difficult to cut government spending beyond some socially acceptable minimum. Debt projections where debt grows eternally violate the government's intertemporal budget constraint, transversality conditions and the fiscal limit.[5]

Responding to criticism, Treasury has attempted to include feedback mechanisms in previous debt projections models. Most notably, Creedy and Scobie (2017) develop a reduced form projections model to capture some of the feedback mechanisms missing from the LTFM.[6] Their model incorporates a limited number of feedback mechanisms, including a risk premium on interest rates that responds to government debt, a savings rate that responds to interest rates, labour effort that responds to tax rates and endogenous productivity which responds to health and education spending. Creedy and Scobie (2017) set their model up to match the exploding debt projections from the LTFM when the feedback mechanisms are switched off. Using their model they find alternative debt paths that return debt to target by the end of the 40 year projection window. While an important step forward, their framework is not general equilibrium in nature, covering a limited number of feedback mechanisms. There is also no requirement that projections from their model represent an equilibrium, as the transversality condition does not have to hold and nor does the intertemporal budget constraint, as demonstrated by their ability to match the LTFM's exploding debt projections. Furthermore, expectations are not modelled and the model is not solved as a system of simultaneous equations, which is key to incorporating the relevant behavioural and feedback channels in general equilibrium. Modifying an earlier version of the fiscal strategy model, a sister model to the LTFM, Fookes (2011) models the fiscal impacts of a magnitude 7.8 earthquake and a prolonged economic downturn on the New Zealand economy. The fiscal strategy model is adapted to include feedback between interest rates and the debt level. Other variables are taken from simulations produced using the New Zealand Treasury model, the Treasury's former forecasting model. This approach is vulnerable to some of the same problems as the Creedy and Scobie (2017) approach.

I take a different tack in this paper, abandoning spreadsheet-style accounting models in favour of a general equilibrium model. More specifically, I develop a reasonably simple stochastic neoclassical growth model to carry out shocks and scenarios analysis. Similar models have been used by Mankiw and Weinzierl (2006), Leeper and Yang (2008), Davig, Leeper, and

3 It is a legal requirement for the CBO to produce projections under current policy settings.

4 The CBO is currently (March 2021) projecting gross government debt in the US to increase from 102 percent of GDP at the end of 2021 to 107 percent of GDP by 2031, and 202 percent of GDP by 2051 (see CBO, 2021).

5 Transversality conditions, in practical terms, state that debt should be equal to the discounted sum of future primary surpluses. Under an exploding debt path, debt will eventually reach a point where it cannot be backed by the discounted sum of the largest primary surpluses a government can run.

6 Ball, Creedy, and Scobie (2015) provide a stochastic version of the model and investigate the probability of exceeding a given debt threshold in each projection year.

Walker (2010), Leeper, Plante, and Traum (2010), Trabandt and Uhlig (2011) and Sims and Wolff (2018) to investigate the short-, medium- and long-run properties of fiscal policy and the macroeconomy, including calculating Laffer curves, calculating fiscal multipliers, carrying out dynamic scoring exercises, modelling the effects of unfunded transfers obligations, and investigating debt financing and debt dynamics. The model I build is plain vanilla in many respects. It shares a number of features with the textbook closed economy growth models of Leeper and Yang (2008), Leeper, Plante, and Traum (2010) and Trabandt and Uhlig (2011). New Zealand is a small open economy, so I extend the textbook closed economy model to the open economy taking the standard approach of Chari, Kehoe, and McGrattan (2002), Galì and Monacelli (2005), Galì and Monacelli (2008) and De Paoli (2009). Following Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020), I augment the model with demographic wedges to try and capture some of the demographic impacts of an ageing population missing from the textbook neoclassical growth model.

Modelling usually involves a number of trade-offs. Additional complexity or detail in some areas may need to come at the expense of simplifying assumptions in other areas. In this paper I have made a deliberate decision not to stray too far from the textbook stochastic neoclassical growth model. This keeps the model relatively simple and tractable, making it easier to understand and explain the results. I do not formally model monetary policy due to the longer projection horizons and conceptual issues regarding natural or potential output on a transition path, although monetary policy responses and proximity to the effective lower bound on interest rates could affect the results in some of the scenarios.[7] The decision to use a somewhat simplified model means that some of the scenarios produced fit better within the framework than others and some scenarios require more judgement to get meaningful results. Government expenditure in the NCGM is more aggregated than the detailed breakdown in the LTFM. As a result the simulations from the NCGM are more stylised than those from the LTFM. Instead I focus attention on trying to capture some of the behavioural and feedback mechanisms absent from the LTFM. The resulting projections from the NCGM are indicative of potential fiscal pressures, the impacts these could have on the wider economy and how policy might react. The NCGM is also a member of a relatively standard family of dynamic macroeconomic models that are used in both public sector forecasting and policy making institutions and academia. This expands the modelling community to which the Treasury belongs and reduces key person risk within the institution.[8]

While the choice of a representative agent model simplifies both the model and its interpretation, it also comes with a cost. In particular the demographic aspects of an ageing population and its impact on the fiscal position can only be investigated at an aggregate level. Shifting demographics will play an important role in the Crown's fiscal position over the next 40 years. In this regard, an overlapping generations model, although more complicated, may be better suited for investigating some of the issues covered in this paper, potentially altering some of the results. That being said, an overlapping generations model also brings more complexity, both in terms of computation, simulation and interpretation.[9] Given this is the first time the Treasury has used a general equilibrium model to produce projections and scenario analysis for the long-term fiscal statement, I have opted for simplicity this time round. However, future research should look into producing long-term economic and fiscal projections using overlapping generations models, with the possibility of using them in supporting analysis for future long-term fiscal statements.

7 To model monetary policy using simple feedback rules usually requires an output gap measure. Because the projections are modelled as a transition path, this would likely mean the calculation and inclusion of the natural rate of output in the model. The natural rate of output would need to be defined and the model augmented with all the natural rate variables that determine the natural rate of output, further complicating the model.

8 Using a model, and a solution and projections methodology that are in many ways "off the shelf" reduces development costs and risks from developing radically new models or methodologies.

9 This may be more so with the Auerbach-Kotlikoff type discrete generations models (see Auerbach and Kotlikoff, 1988). The Baksa, Munkacsi, and Nerlich (2020) extension of the Gertler (1999) model (see also Baksa and Munkacsi, 2016), itself an extension of the Blanchard-Yaari continous generations OLG model, may be a computationally cheaper model to solve, providing a more flexible alternative in this regard.

For the first time, the Treasury is considering the impact of climate change on the Crown's fiscal position in the long-term fiscal statement. As part of this effort, I undertake modelling work to support the long-term fiscal statement. While the NCGM does not contain all the necessary variables, sectors and policy levers to investigate mitigation policy and the full impact of adaptation, I am able to investigate the impact of weather shocks, such as droughts and storms. As the effects of climate change become more pronounced over the 40 year reporting window, droughts are predicted to increase in frequency and duration, while storms are predicted to increase in intensity. I model droughts as negative productivity shocks, an approach used by Gallic and Vermandel (2019) in a similar model. The drought scenarios are calibrated to match existing evidence (Kamber, McDonald, and Price, 2013) on their economic impact. The economic impact of storms is modelled using capital destruction shocks, an approach that has been more widely used to model natural disasters. I develop a method of producing stochastic simulations, where extreme weather events are modelled as discrete events that arrive according to a Poisson process. This enables the fiscal implications of changes in both the intensity and frequency of extreme weather events to be investigated over the reporting period.

The ageing population and climate change scenarios are kept separate for both clarity and practical purposes. I produce a baseline ageing population scenario by matching the NCGM's non-superannuation spending paths, as a share of GDP, with their counterparts in the LTFM. The superannuation spending path is constructed using demographic projections and the NCGM's endogenous wage path. Dynamic equilibrium in the NCGM requires at least one of government's fiscal instruments to respond with sufficient strength to deviations of debt from its target to ensure debt's stationarity. To achieve this, I assume government raises tax rates on labour and capital income, and consumption expenditure reasonably aggressively to try and balance the budget in each period and stabilise net debt around its target. In the baseline projection, net debt as a share of GDP is kept reasonably stable around its target over the 40 year reporting period, but this requires substantial increases in tax rates, resulting in a GDP path that is 3.7 percent lower than what it would have been in the absence of increases in taxes and government expenditure.[10]

The baseline projection is built on a number of key assumptions. I produce a range of alternative scenarios where some of these assumptions are altered or the model is subjected to economic and physical shocks, to see how the results change. In particular I look at how restraining superannuation and health spending, separately, affects the results. I test fiscal resilience to economic and physical shocks by constructing recessions and earthquake scenarios, separately, on top of the baseline ageing population scenario. I also look at how both fiscal consolidation undertaken at different speeds and the delayed stabilisation of debt in the face of increasing spending pressures affect the results. The main results from these scenarios can be summarised as follows:

- *Spending Restraint*: Restraining either health or superannuation spending means taxes do not need to increase by as much to balance the budget. Because taxes are distortionary, smaller increases in taxation lead to smaller declines in GDP relative to the ageing population baseline. Restraining superannuation spending results in smaller GDP losses because there is a partial offset from increased health spending due to the labour supply effects from the negative wealth effect.
- *Recessions*: A series of recessions, calibrated to broadly match historical stylised facts, are added to the baseline projection. They raise net debt to GDP by between 11 and 13 percentage points. A reasonably aggressive tax response is required from government in order to bring net debt back to target before the next recession hits.

10 The average tax rate on labour income needs to increase by 6.4 percentage points, the average tax rate on capital income needs to increase by 10 percentage points, while the average tax rate on consumption expenditure needs to increase by 5 percentage points.

- *An Earthquake*: A large earthquake, calibrated to match what a large earthquake in Wellington could look like, raises net debt to GDP by 12 percentage points, similar to the recessions scenario, taking more than a decade to return to target. The overall fall in GDP is smaller compared with the recessions scenario, due to the faster decline and recovery in trend total factor productivity and the rebuild leading to large scale increases in private and public investment, which both factor in a healthy degree of "building back better".

- *Fiscal Consolidation*: Fast fiscal consolidations are more costly (in a GDP sense) in the short run, falling on a narrower cohort of tax payers and government service users, but generate more benefit in the long run as debt servicing costs are reduced. Slow fiscal consolidations spread the economic costs over a longer period of time, so that a wider cohort of tax payers and government service users experience smaller per period costs over a longer period of time. Ultimately a longer period of elevated debt in the slow consolidation scenario translates into higher tax rates, lower spending and a larger cumulative GDP cost.

- *A Delayed Response*: Delayed fiscal responses to increasing debt generate benefits in the short run, as additional government spending increases aggregate demand without the costs of higher taxation. However, when government decides to stabilise debt at a higher level, or consolidate, taxes need to be raised rapidly to fund higher debt servicing costs and the higher primary surpluses required to bring debt back to target. These higher taxes reduce economic activity and the level of GDP. Overall, the net gains from delaying, measured by the cumulative undiscounted output gap, are negative in the long run in most of the scenarios investigated.

I produce separate stochastic scenarios that investigate the fiscal and economic implications of changes in the frequency of droughts and the intensity of storms due to the effects of climate change. In the droughts scenario, droughts are assumed to increase in frequency with current 1 in 10 year and 1 in 20 year events becoming twice as likely by 2040 and three times as likely by 2060. My simulation results suggest that the increase in drought frequency could lower GDP by 0.5 percent relative to trend, on average, by the end of the projection period, while net debt to GDP could be 1.2 percentage points higher on average. In the storms scenario, I assume that severe storms are expected to arrive once every 10 years, and increase in intensity to destroy 2 percent of the capital stock by the end of the reporting period. My simulation results suggest that GDP could be 0.7 percent lower than trend, on average, by the end of the reporting period and net debt to GDP could be 3 percentage points higher, on average. I note that droughts and storms add a measurable amount of volatility to both debt and GDP.

The overall fiscal impact of the weather events simulations is smaller than the fiscal impact of the ageing population. However, caution must be taken when interpreting these results. In particular, I am only modelling a very narrow range of aspects related to climate change, so my results shouldn't be interpreted as capturing all the effects of climate change. I do not take into account the effects of mitigation or adaptation, nor do I take into account how climate change and climate change policy may affect trend productivity or the world economy, which could have larger impacts on the New Zealand economy through trade and migration.

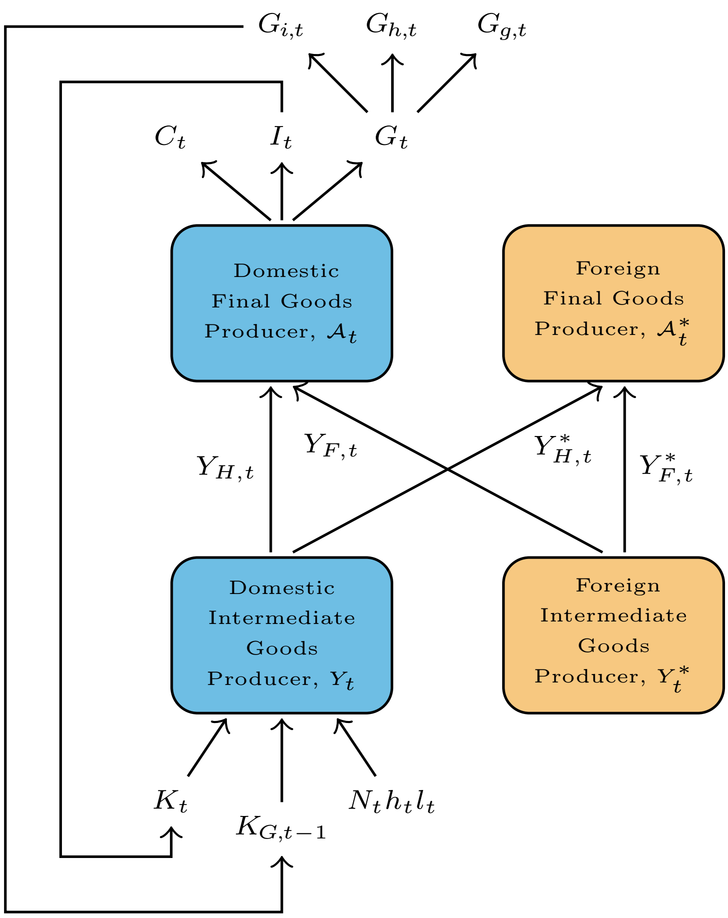
The rest of the paper is structured as follows; Section 2 gives a brief description of the model (a full derivation and description can be found in Appendices A through E). In Section 3 I compare and contrast the NCGM with the long-term fiscal model and describe the relationship between the models. In Section 4 I discuss how the model has been calibrated, solved and some of the assumptions made about key exogenous variables. In Section 5 I present the model simulations and scenarios for an ageing population, and in Section 6 I present the impact of extreme weather events that increase in size and frequency with climate change. Section 7 concludes.

2. The Model

The model I develop for use in this paper is plain vanilla in many ways. It shares a number of features with the textbook closed economy stochastic neoclassical growth models of Mankiw and Weinzierl (2006), Leeper and Yang (2008), Leeper, Plante, and Traum (2010) and Trabandt and Uhlig (2011). New Zealand is a small open economy so I extend the model to the open economy following the textbook approach of Chari, Kehoe, and McGrattan (2002), Galì and Monacelli (2005), Galì and Monacelli (2008) and De Paoli (2009). I incorporate the effects of an ageing population into the model by introducing demographic wedges as described in Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020).

The model economy is populated by a representative household, a representative intermediate goods producer, a representative final goods producer, a fiscal authority and a foreign economy. The inclusion of government debt and an intertemporally-balanced government budget means fiscal policy plays an important role stabilising net government debt around its long-run target. I describe the model structure using two diagrams: one that covers the model's production structure, and the other covering the flows between the different actors in the model. A full mathematical derivation and description of the model can be found in Appendices A through E. Figure 1 illustrates the production structure in the NCGM, where variables are defined in aggregate terms.

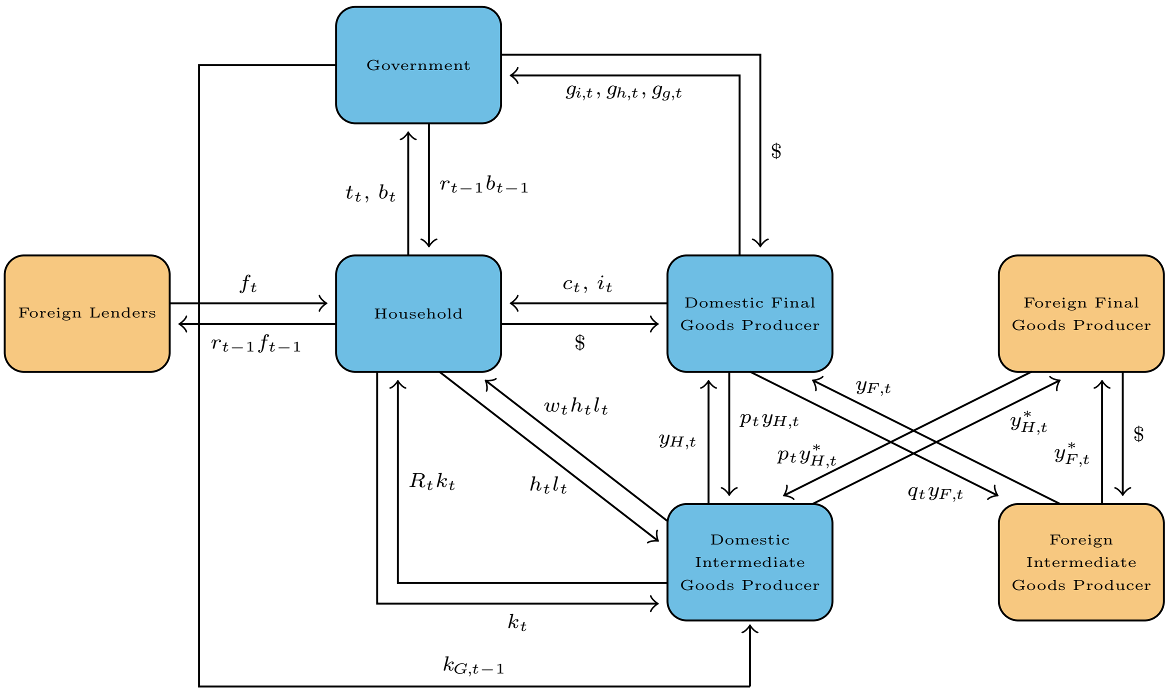
Figure 1 — Production Structure



Domestic intermediate goods, Y_t, are produced using effective private capital, K_t, effective labour, $N_t h_t l_t$, and government capital $K_{G,t-1}$. Domestic intermediate goods are either used in the production of domestic final goods, $\mathcal{A}_t$, or exported and used in the production of foreign final goods, $\mathcal{A}_t^*$. Domestic final goods are produced using domestic intermediate goods, $Y_{H,t}$, and imported intermediate goods, $Y_{F,t}$. Domestically produced final goods can either be consumed, C_t, invested, I_t, or used by government, G_t. Investment goods become part of next period's capital stock. Government goods are further split into investment, $G_{i,t}$, spending on health, $G_{h,t}$, and general government expenditure, $G_{g,t}$. Government investment becomes part of next period's government capital stock.

Figure 2 is a flow diagram depiction of the NCGM, where variables are defined in per capita terms.

Figure 2 – Model Flow Diagram



The representative household owns the capital stock, k_t, and supplies effective labour, $h_t l_t$, and capital services to the representative domestic intermediate goods producer. In return the household receives labour income, $w_t h_t l_t$, and dividends, $R_t k_t$. The representative domestic intermediate goods producer produces an intermediate good, y_t, that is consumed by both domestic ($y_{H,t}$) and foreign final goods producers ($y^*_{H,t}$). In return they receive payment equal to $p_t y_t = p_t(y_{H,t} + y^*_{H,t})$. The representative final goods producer produces a final good, a_t, using both domestically produced and foreign produced intermediates, y_t^*.[11] The final good can either be consumed (c_t) or invested (i_t) by the representative household, or invested ($g_{i,t}$), spent on health ($g_{h,t}$) or general government spending ($g_{g,t}$), by the government. Government finances their spending by either borrowing from households, b_t, or raising taxes on households, t_t. Government repays debt to households with interest, $r_{t-1}b_{t-1}$. Households can borrow from abroad, f_t, repaying the debt with interest, $r_{t-1}f_{t-1}$.

11 Technically the payment to the foreign intermediate goods producer should be $p_t^* y_t^*$ instead of \$. However, I assume the share of the domestic input in the production of the foreign final good is extremely small, so that the price of the final good is equal to the price of the intermediate good in the foreign country.

3. A Comparison with the Long-Term Fiscal Model

I compare and contrast the NCGM with the long-term fiscal model (LTFM), and explain the relationship between the models. The LTFM is a large spreadsheet accounting model that is used to construct government spending projections at an annual frequency over a 40 year horizon. It has been used to construct the health and superannuation spending tracks that form the baseline ageing population scenario for the long-term fiscal statement. It contains both historical data and the Treasury's latest 5 year fiscal and economic forecasts. The LTFM has a lot of detail and disaggregation on the government spending side, with government spending broken into a number of components, including health, education and superannuation. Earlier iterations of the LTFM were used to construct the central projections for the ageing population scenarios for previous editions of the long-term fiscal statement. Debt projections are constructed from the model under the assumption that tax policy remains unchanged. Without making any policy changes, the LTFM shows that government debt explodes when current government spending patterns, in combination with the ageing population, are projected forward. The LTFM's debt projections lead to the conclusion that fiscal policy under current settings is not sustainable and that changes need to be made to policy to ensure its sustainability.

While the LTFM is a useful tool, like all models, it has its limitations. Because it is a spreadsheet accounting model, it does not capture some of the key behavioural responses and feedback mechanisms in the economy. On top of this, the exploding debt paths it produces do not represent an equilibrium. Dynamic equilibrium requires the transversality condition to hold. The transversality condition, which prevents government from continuously borrowing to repay its debt, implies that debt must be equal to the discounted sum of expected primary surpluses, which means at some point in the future increases in government expenditure must be backed by increases in tax revenues (see Doepke, Lehnert, and Sellgren, 1999, for example). When debt is expected to follow an explosive path, both the transversality condition and the government's intertemporal budget constraint are violated. Furthermore, it is unlikely that debt would get to these levels. Government would either change policy to prevent debt getting this large, or default once it could no longer repay its debt. As Leeper (2010) points out, politics and/or the Laffer curve place limits on the amount of tax that can be raised, just as there are limits to how much government spending can be cut, which together place a fiscal limit on debt. If lenders believed government debt was going to explode and that government would not be able to redeem its debt upon maturity, they would no longer hold government debt. The fact that lenders hold government debt means they believe they will be able to redeem the debt at some point in the future.

The NCGM is a dynamic general equilibrium model. Its microfounded behavioural equations represent the solutions to the household's and the firms' utility and profit maximisation problems. It is a forward looking simultaneous equations model that is solved under perfect foresight in the baseline scenario and under rational expectations in the stochastic scenarios.[12] Both of these solution methods require the endogenous variables in the model to satisfy dynamic equilibrium. Government spending, transfers spending and tax policy must satisfy the government's budget constraint, and the government's intertemporal budget constraint holds in expectation. Equilibrium requires the transversality conditions to hold, so that on average debt does not grow faster than real interest rates. More importantly at least one of government's fiscal instruments must respond to deviations of government debt (or the primary balance) from its long-run target

[12] While rational expectations is the most common assumption used to model expectations in dynamic stochastic macroeconomic models, it is not a defining or limiting characteristic of these models as discussed by Ghironi (2017). Alternatives to rational expectations exist including, rationalisable equilibrium dynamics, restricted perceptions equilibria, decreased-gain and constant-gain variants of least-squares learning, rational belief equilibria and near-rational expectations, to name a few (see Woodford, 2013, for a survey and applications.). That being said Marcet and Nicolini (1997) describes the dilemma: rational expectations are too demanding of agents while moving away from rational expectations can lead to the "jungle of irrationality".

with sufficient strength to ensure that a dynamic equilibrium exists. The additional structure, in combination with the inclusion of agents that respond to their economic environment and the general equilibrium nature of the model, means I am able to capture some of the behavioural responses and feedback mechanisms that are not present in the LTFM. However, in order to build in more structure and include the household's and firms' behavioural responses I have sacrificed some detail on the fiscal side of the model. As a consequence, the projections from the NCGM are more stylised than those from the LTFM, but at the same time they provide a useful illustration of how the economy responds to fiscal policy and how fiscal policy responds to the economy. On top of this, the NCGM is stochastic, and can be solved under the assumption of rational expectations when shocks are present. This means the model can be shocked in an internally consistent manner, which makes the model particularly useful for investigating the impact of shocks and uncertainty on the long-run fiscal position.

One of the key feedback mechanisms that is missing from the LTFM is the endogenous response of market interest rates to the level of government debt. The domestic real interest rate in the NCGM is endogenous and is determined in equilibrium as the level that ensures the supply of debt is equated with the demand for debt. An ad hoc risk premium is attached to the household's borrowing from abroad. The household effectively acts as an intermediary for government, borrowing more from abroad when they cannot meet the government's demand for debt. This risk premium serves a dual purpose: it raises the interest rate paid on both net government debt and net foreign debt when net foreign debt is above its steady state/balanced growth path level, reflecting the additional risk accompanied with higher debt. It also serves a mechanical function, stationarising net foreign debt (see Schmitt-Grohe and Uribe, 2003, for a discussion of closing small open economy models). The risk premium is ad hoc, because it is imposed on the model and is not generated endogenously from the solution of the model.[13] I do not explicitly capture default risk in the model or the solution method. Handling default risk in similar models is at the current research frontier (see Bi, 2012, for example) and requires more computationally intensive solution algorithms, which place restrictions on the size of the model and the type of analysis that can be performed.

Another key difference between the NCGM and the LTFM is the "distortionary" impact taxation has on agents' behaviour. In the NCGM taxes on capital income, labour income and consumption alter the incentives the representative household faces when making decisions. For example, an increase in the tax rate on labour income reduces the overall return from working, all else equal, reducing the quantity of labour supplied. Similar considerations apply to a change in tax rates that affect the returns from capital income. Therefore, just as a reduction in tax rates is expected to stimulate economic activity, so should an increase in tax rates be expected to dampen economic activity. This means that raising tax revenue is not a simple transfer of resources from households to the government. Instead, a loss in economic activity purely through the mechanism of raising taxes should be expected. In practice, the size of this loss depends on the magnitude of the tax changes as well as their incidence.[14]

The overall impact of changes in taxation in the NCGM also depends on how that revenue is used. A portion of total government revenue is spent on "government investment", which acts

[13] Fully stochastic non-linear solution algorithms for rational expectations models will usually generate an endogenous risk premium due to Jensen's inequality.

[14] With regards to labour taxes, Creedy et al. (2018) show that even small changes in labour supply as a result of higher taxation are consistent with large economic costs. The wider empirical literature suggests that company and personal income taxes (both of which tax capital income) are more negative for economic efficiency and growth than other taxes such as consumption and property taxes (OECD, 2010), and a number of studies have suggested that capital taxation is more distortionary than labour taxation (e.g. Coleman (2019) provides a review). Also Implicit within the NCGM is the assumption that the effects of changes to taxation on GDP are non-linear. That is, the responsiveness of GDP to tax changes depends on the initial level of tax rates. So for example, the higher the initial tax rate, the greater the economic impact of a proportionate tax increase or tax cut. This assumption is broadly supported by the literature (e.g. Creedy (2003) and Auerbach (1985)).

as a productive input to the firm's output. Government consumption is assumed to be non-productive, although it contributes to aggregate demand, while government transfers flow back to the household as income, and so can be spent or saved. The government consumption and investment channels provide some offset to the distortionary impacts of taxation. However, the NCGM does not distinguish between different household types, and so there is no redistribution of resources between households with different propensities to consume. Importantly, government consumption of goods and services does not have a direct supply side impact.[15] Rather, it is assumed that government goods and services provide a social benefit that is not captured in the model. This is a simplifying assumption, which misses an aspect of the stimulatory effects of fiscal policy. Finally, the principal focus of the NCGM is to shed light on the effects of fiscal policy on income and output. Ultimately, the net impact of taxation and spending should be assessed in terms of the wider effects on living standards.

The LTFM is a necessary input for the NCGM. Some of the projected paths from the LTFM inform the baseline for the NCGM. Due to the assumptions used to construct and solve both models, I will not be able to match all the tracks from the NCGM with the LTFM, and nor should I try to. Each model has their own relative advantages and merits and they should be viewed as complements. The LTFM provides an important tool for calculating the government and transfers spending tracks that are used as the baseline in the NCGM. It also provides a useful benchmark for what the economy would look like without the behavioural responses and feedback mechanisms present in the NCGM.

[15] Government consumption affects the supply side indirectly through the income/wealth effects on labour supply through crowding out, and through changes in taxation required to fund the changes in government consumption.

4. Parameterisation, Solution Method and Exogenous Assumptions

In this section I discuss the model's calibration, how the model is solved, how the projections are produced, and the assumptions made about a key set of exogenous variables. The model is calibrated to match some short- and long-run properties of the New Zealand economy. In particular I choose some of the model's parameters to match great ratios and historical data averages, with the remaining parameters taken from the literature. A full list of the parameters, their baseline values, and how they are set can be found in Appendix F.

Producing the projections and scenarios for the LTFS requires a modelling methodology that can incorporate both long-run/slow-moving demographic trends occuring over decades and the ability to model stochastic scenarios (shocks and stress testing exercises) which have implications for the short and medium run. This presents some challenges and requires some trade-offs to be made. To meet these challenges, I model the baseline scenario as a deterministic transition path, and I model the stochastic scenarios on top of the baseline using the (stochastic) extended path algorithm.

The baseline projection is modelled as a deterministic transition path from an initial steady state to a terminal steady state, subject to the future paths of the exogenous variables. Perfect foresight is assumed, which means agents are surprised in the first projection period by the entire future path of the exogenous variables, which are then perfectly anticipated in all subsequent projection periods.[16] The initial steady state is chosen to roughly match current conditions and it is assumed that the economy has been stationary at this level for a number of periods before the transition starts.[17] The terminal steady state is chosen to match a future point in time, beyond the 40 year reporting horizon, where the exogenous variables are all assumed to have stabilised at a constant level. The derivation of the initial and terminal steady states is described in Appendix C and Appendix D, respectively. I set the time frequency in the model to quarterly for all projections because many of the stylised facts that underpin the scenarios are based on quarterly data,[18] and I set the projection horizon to 2000 periods which should be long enough to ensure convergence to the terminal steady state.[19] The transition path is then solved by stacking the non-linear model equations for each time period into a vector, plugging in the paths for the exogenous variables and the values for both the initial and terminal steady states, and then iterating using Newton's method until convergence is achieved.

Typically in exercises like these, where slow moving changes are expected to occur over a long period of time, the future is modelled as a transition from an initial steady state, chosen to be close to current economic conditions, to a terminal steady state, where the exogenous variables

[16] In the first projection period agents are surprised by changes in government policy. In the absence of shocks government commits to spending policies for the entire projection period, including the terminal steady state.

[17] The projections could have been started in an earlier time period, rather than using the current time period, to lessen the frontloaded response from agents being surprised by the entire future path of exogenous variables and to account for a demographic transition that has already been underway for decades. This is the approach taken by Bielecki, Brzoza-Brzezina, and Kolasa (2018). While conceptually appealing this approach may be inconsistent with current fiscal policy, which has not committed to fully funding superannuation for those aged over 65 years or the projected increases in health expenditure out to 2061. The COVID shock would also make this approach trickier to implement.

[18] For example, it is easier to quantify the properties of recessions using quarterly data because the definition of a recession uses quarterly data.

[19] The projection horizon needs to be long enough that the model economy has reached the terminal steady state by this point. On the other hand a shorter projection period potentially speeds up the algorithm as the dimensions of the problem are smaller. Ideally the projection horizon would be chosen to be the exact period that the economy reaches the terminal steady state, but this is not known in advance, and it will typically vary from scenario to scenario, so I opt for a projection horizon that is on the larger side. After each model run, I check the model has reached the terminal steady state by the end of the projection period.

are assumed to have stabilised, although this is not a requirement. In normal times when the shocks are small, starting from a steady state close to current economic conditions seems reasonable. However, the COVID shock is large, which means the economy is likely to be further from the steady state than would be ideal. It may be more reasonable to choose initial conditions that represent deviations from an initial steady state generated by a shock or a combination of shocks, although this too presents challenges. In particular, the model has a large number of endogenous variables which makes the coherent choice of initial conditions more difficult with many of the model variables not included in the typical set that Treasury regularly monitors or forecasts.[20] Furthermore, the shocks and shock processes would need to be fine tuned to ensure plausible looking transition dynamics, and there are no guarantees these judgements will look reasonable across all scenarios.[21] Given many of the fiscal challenges facing New Zealand are long run in nature, and would be the same regardless of COVID, I simplify the analysis by starting the projections from the steady state. This means a number of simplifying assumptions need to be made, and some of the cyclical impact of COVID needs to be removed from the initial conditions.

A number of variables are treated as exogenous in the projections. That is I specify a path for these variables that is not determined by any of the model equations. Permanent changes in these exogenous variables that occur over the projection period cause the model economy to shift from the initial steady state to the terminal steady state. Specifically, I treat the following variables as exogenous:

- The population growth rate
- The proportion of the population over the age of 65 (the old-age dependency ratio)
- The exogenous component of the aggregate disutility of working[22]
- The level of foreign real GDP
- The foreign real interest rate
- The ratio of health spending to GDP
- The ratio of government investment to GDP
- The ratio of general government expenditure to GDP
- The ratio of transfers spending to GDP
- The capital tax rate target
- The labour tax rate target
- The consumption tax rate target

Some of these variables are exogenised because they reflect expected demographic changes over the next 40 years, which are taken as given. Some of these variables are exogenised because they represent policy assumptions and the policy adjustments required to maintain equilibrium. The remaining variables are treated as exogenous because they are determined

[20] If the model were linearised, richer in dynamics, frictions and shocks, then the Kalman filter could be used to find a set of initial conditions, represented as deviations from the steady state that match current initial conditions. Although this too would most likely require detrending the data introducing additional conceptual issues about what is being matched. Furthermore, doing this correctly would require at least as many shocks as there are initial conditions to be matched.

[21] Treasury produces regular forecasts for the BEFU and the HYEFU and it is not clear that these should be matched either, given they are produced under alternative policy and demographic assumptions. Nor would it be fair to change the shock processes between scenarios when it is the policy responses that are the main focus of the exercise.

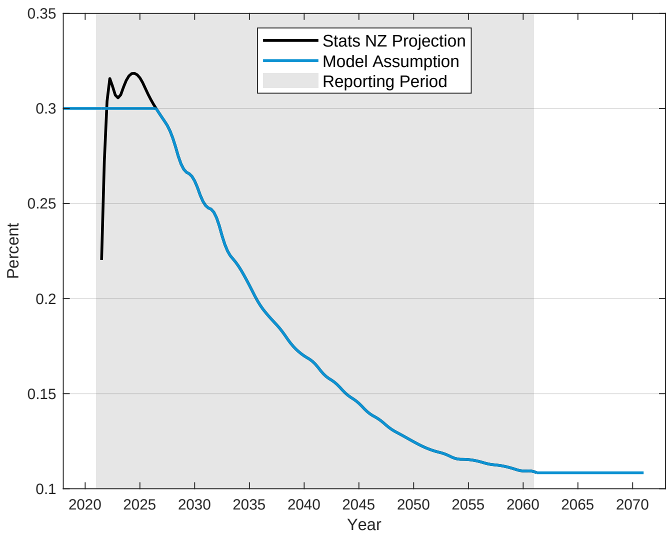
[22] This represents aggregate preferences for working, which are assumed to change with the composition of the population.

outside New Zealand, where New Zealand is assumed to be a small open economy, too small to have a material impact on their determination.

Both the perfect foresight and the extended path algorithms require a terminal steady state, a point in the future where the exogenous variables have stabilised at a constant level. I assume that the exogenous variables stabilise around their 2062Q2 values, which in practise means exogenous variables beyond 2062Q2 are set at their 2062Q2 values. This is a somewhat arbitrary decision that has consequences for the results. Setting the date just outside the 40 year reporting window avoids sharp responses in the forward looking variables as agents anticipate the stabilisation of both the exogenous variables and the economy at the terminal steady state.[23] Pushing the stabilisation date for the exogenous variables further into the future would likely worsen the economic projections as forward looking agents in the model would expect continued increases in health and superannuation costs beyond the 40 year reporting horizon, requiring even higher distortionary tax rates to balance the budget.

The projected paths for the population growth rate and the proportion of the population over the age of 65 are driven by demographic factors not determined by the model. Over the 40 year reporting horizon these are set equal to the Statistics New Zealand projections for the same period. The Statistics New Zealand projections, reported at an annual frequency, are interpolated using a cubic spline to fit the quarterly NCGM. From 2062Q2 onwards these variables are fixed at their 2062Q2 values, allowing the demographic transition to be modelled as a transition path. For population growth the stabilising assumption is relatively innocuous, and leads to the population growth track presented in Figure 3 being used in the model,

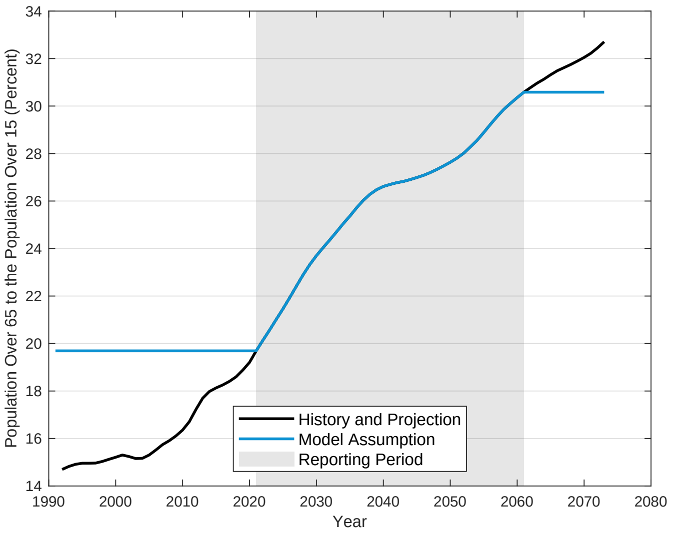
Figure 3 – Projected Population Growth (Quarterly)



I set the first 20 quarters of population growth to 0.3 percent to remove the impact that short-term volatility and the lower initial condition may have on the results. For the proportion of the population over the age of 65 (the old-age dependency ratio), I use the track presented in Figure 4 in the model,

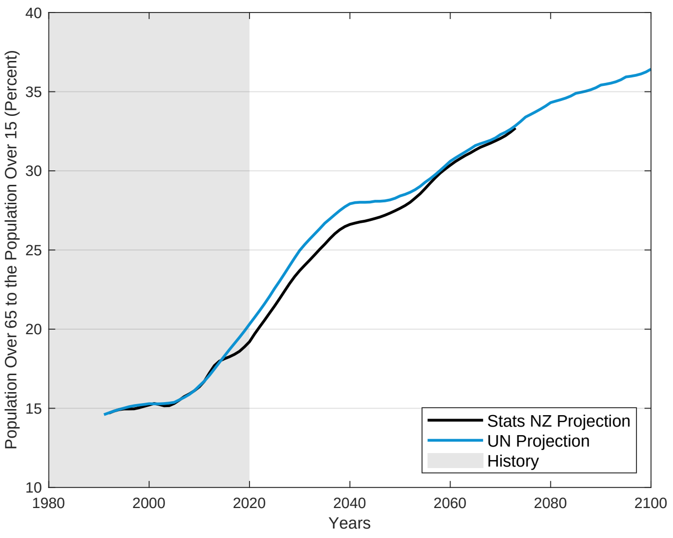
[23] Stabilising the exogenous variables at a particular date can create sharp corners on some of these variables, especially when these variables are growing rapidly over the projection period and are suddenly stopped. This can have consequences for agents' behaviour as they near this point in time. Pushing this point just outside the reporting window hides some of this behaviour. Alternatively, some of the sharp corners could be smoothed by gradually transitioning from the projection to a constant level, although this would also require some arbitrary judgment and would probably need to occur outside the reporting window.

Figure 4 — Projected Old-Age Dependency Ratio



In the modelling scenarios I assume that the proportion of the population over the age of 65 stabilises in 2062Q2 and subsequent periods at 2062Q2 levels. This is a simplifying assumption which allows the demographic transition to be modelled as a transition path. However the proportion of the population over the age of 65 is expected to continue growing past the 2062Q2 level chosen to represent the terminal steady state. In fact, United Nations projections (see Figure 5) suggest the proportion of the population over the age of 65 is expected to continue growing past the year 2100.[24]

Figure 5 — Projected Old-Age Dependency Ratio

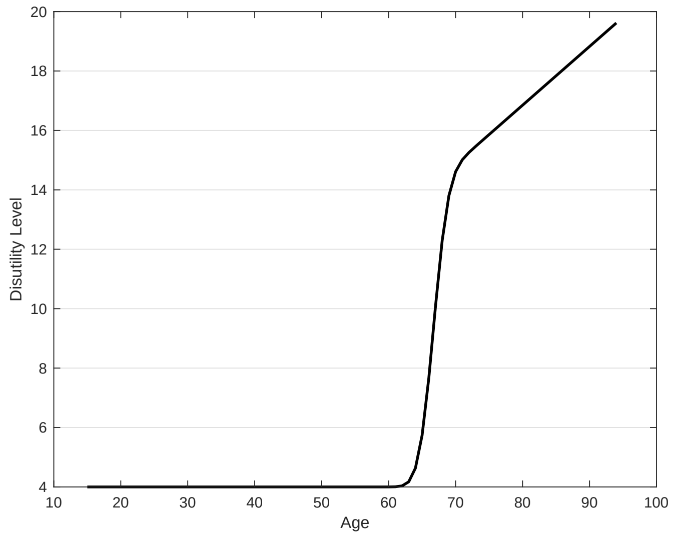


Total superannuation payments in the model, in per capita terms, are directly linked to the proportion of the population over the age of 65. By stabilising the proportion of the population over 65 around its 2062Q2 levels, I am assuming that government credibly commits to funding superannuation payments to those over the age of 65 until 2062Q2 and then adjusts the superannuation age of eligibility to ensure that the proportion of the population eligible for superannuation remains constant at 2062 levels beyond 2062.

24 This highlights the fact that this is largely driven by improvements in life expectancy and reductions in fertility rates, with the trend continuing long after the baby boomers are gone.

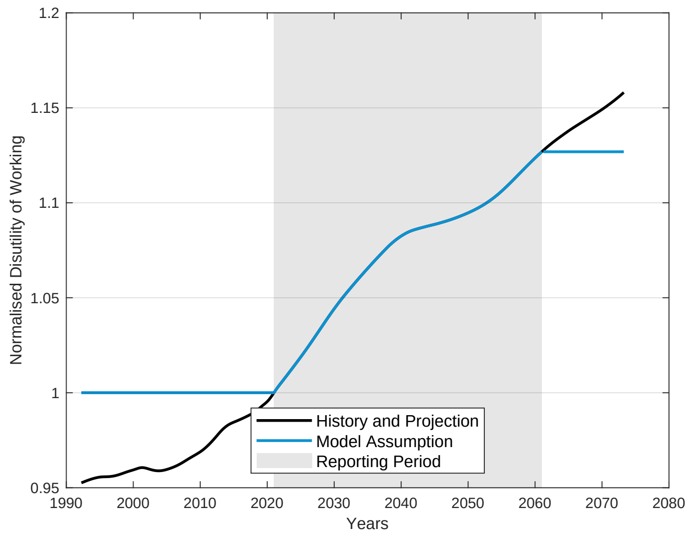
It is common in overlapping generations models (and a number of representative agent models that approximate OLG models) to assume that older workers place a higher value on leisure than their younger counterparts. Following Jones (2018) and Papetti (2019), I incorporate this feature into the model projections by making the aggregate distutility of working a function of both the age-related disutility of working and the age structure of the population. As a result, changes in the age structure of the population will be reflected in changes in the aggregate disutility of working, so that an ageing population will have implications for labour supply. Following Jones (2018) and Papetti (2019), I use the age-related disutility of working profile from Kulish, Kent, and Smith (2010) which is plotted in Figure 6 below, to represent workers age-related preferences for working.

Figure 6 — Age-Related Disutility of Working



Combining the age-related disutility of working, with the projected age structure of the population,[25] I obtain a path for the aggregate disutility of working, which I present in Figure 7,

Figure 7 — Aggregate Disutility of Working



25 This is done according to equation (285).

My modelling assumptions imply that the exogenous component of the disutility of working increases by 13 percent over the 40 year reporting period. The disutility of working will continue to grow beyond the 40 year reporting horizon as the composition of the population continues to change. However, consistent with modelling this problem as a transition path, I assume that the aggregate disutility of working stabilises around its 2062Q2 level after 2062.

There are two foreign variables in the model, foreign GDP and the foreign real interest rate. Both variables are assumed to represent a trade weighted aggregate of New Zealand's main trading partners. Foreign GDP affects the domestic economy through the demand for exports. The foreign real interest rate affects the real exchange rate through a modified real (uncovered) interest parity condition. Due to the uncertainty around the expected future paths of these variables, I keep both the foreign real interest rate and the foreign level of GDP (in effective units of labour) constant, to simplify the analysis.[26]

A number of policy variables and responses are also exogenised, capturing external spending pressures and the tax responses required to maintain equilibrium. In the baseline NCGM projection I assume the same spending pressures as the LTFM with tax rates adjusting to approximately balance the budget. In practice this means matching the ratios of government spending and non-superannuation transfers spending to GDP in the NCGM with their counterparts in the LTFM. Matching the ratios rather than the levels implicitly captures the linkage between labour costs (a large direct and indirect component of government spending), labour productivity and the level of GDP. Furthermore, many of the government expenditure components have remained reasonably constant as a share of GDP over history. It then follows that changes in productivity growth will only have a small effect on the results.[27] On the flipside, matching ratios means that more judgement needs to be applied in some of the stochastic scenarios to capture realistic automatic stabilisers.

Modellers and practitioners typically construct long-run health spending projections by i) extrapolating historical trends, ii) using general equilibrium models, or iii) using micro-simulation models (see NRC, 2010, for further discussion). The health track used in the NCGM is matched to the LTFM, where the LTFM's approach can be characterised as extrapolative. The health track in the LTFM is constructed as a function of a weighted demographic factor (capturing the ageing population), the inflation rate, labour productivity, a healthy ageing factor and health productivity, as described in Bell (2021). On top of this a non-demographic growth factor is included to take into account the fact that health costs have grown much faster than the other factors would have predicted. The health projections are then constructed from projections of all the factors including the non-demographic growth factor which is extrapolated over the reporting period using the growth factor that best fits recent history. Most of the increase in health spending as a share of GDP is driven by this non-demographic growth factor. Much of the US literature refers to health projections as GDP + x, indicating that health expenditure is expected to grow at the rate of GDP plus some excess rate x. Historically the Centers for Medicare and Medicaid (CMS) in the US has used GDP + 1 to make projections for health care costs, based on the observation that US health care spending has grown at a rate that is 1 percentage point faster than GDP growth. Matching the health spending to GDP ratio from the NCGM to the LTFM is equivalent to specifying a health spending track as some growth rate x over and above the GDP growth rate, where x is largely driven by the non-demographic growth factor. A drawback of this approach is it

26 This implies that the foreign economy grows at the same rate as domestic total factor productivity and the domestic population. Assuming differences in productivity or population growth rates would introduce a more complicated transition path, or require more flexible Cobb-Douglas aggregation of the final good, which would have trend implications for the real exchange rate.

27 This will mainly be through the obsolescence channel for capital goods and its impact on the capital output and investment output ratios. The assumption that expenses will increase with productivity growth means increased productivity growth is not expected to be a large help for the fiscal challenges facing the nation.

does not specify an asymptote for health spending as a share of GDP.[28] More recent modelling work by the CMS using a neoclassical growth rate builds in an asymptote for health spending, based on household preferences, although this approach may be more natural for a system like the US where consumers have more choice over their healthcare expenditure.[29]

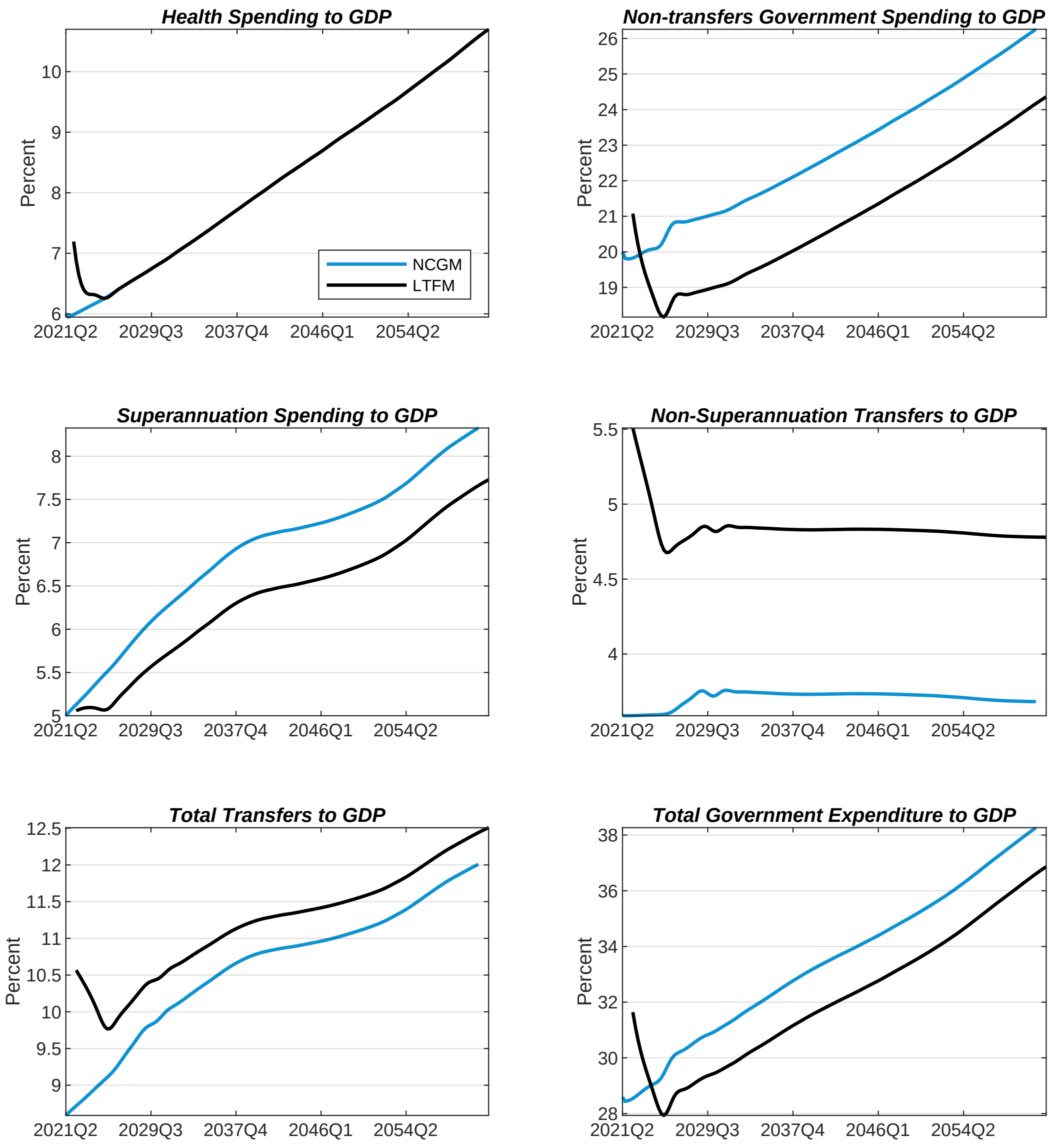
Matching the government spending and non-superannuation transfers tracks from the NCGM with the LTFM presents a few difficulties. In particular, differences in the data concepts, the deviations from trend caused by the COVID shock and the lack of a steady state or balanced growth concept in the LTFM, means that some adjustments need to be made to produce an NCGM projection that accurately captures the long-term expenditure pressures in the LTFM. The NCGM is a more stylised representation of the economy and the data than the LTFM, and has been calibrated to broadly match national accounts data in steady state (on a balanced growth path) although consideration is taken for Crown accounts data and concepts. By contrast, the LTFM matches Crown accounts data exactly, as it is required to do so. National accounts measures of non-transfers government spending form a larger share of GDP than Core Crown non-transfers government spending. Furthermore the projections begin shortly after one of the largest economic shocks in living memory, which means the economy is almost certainly some distance from the steady state.[30] To capture the spending pressures built into the LTFM projections I match the increase in the GDP shares of transfers and general government spending over the post forecast reporting period (2026 to 2061), adjusting for the difference in the initial period. For health spending I match the ratio to GDP from the NCGM with the LTFM over the 2026 to 2062 period. I backdate the projection over the 2021 to 2026 period, which gives an initial condition of 6 percent, not too far from the 5.9 percent recorded in the 2018/2019 financial year. For the other spending variables (excluding superannuation spending, which is endogenous) I match the increase in the share of GDP over the 2026 to 2062 period. I then backdate the variables over the 2021 to 2026 period. An adjustment is made to the initial condition to better match the national accounts data and the restrictions that starting the projections from a steady state imposes. For non-transfers government spending to GDP, this results in a track that is approximately 2 percentage points higher, but parallel to the LTFM track over the 2026 to 2062 period. For non-superannuation transfers, this results in a track that is about 1 percent point lower, but parallel to the LTFM track over the 2026 to 2061 period. So even though the initial starting value for total government expenditure as a share of GDP is different, I capture the increase in the share over the 2026 to 2062 period. Figure 8 shows the spending tracks from the NCGM against their counterparts in the LTFM.

28 Some earlier extrapolative projections had total healthcare spending reaching 99 percent of GDP by 2080 in the US (see NRC, 2010).

29 Borger, Rutherford, and Won (2008) construct a simple neoclassical growth model, not too different from the core of the model developed in this paper, and use it to produce health projections for the US going out 75 years. Health expenditure is determined endogenously and depending on the parameterisation of the model, health spending eventually asymptotes as health costs become too high. Attanasio, Kitao, and Violante (2010) also build endogenous health spending into a general equilibrium model.

30 A steady state total tax revenue to GDP ratio, combined with a steady state net debt to GDP ratio (debt target) along with the steady state real interest rate/growth rate differential pins down the steady state government spending to GDP ratio. In the calibration of the initial steady state, the non-superannuation transfers to GDP ratio is determined as the residual. Lowering the level of steady state non-transfers government spending as a ratio of GDP implies a higher private consumption to GDP ratio.

Figure 8 — Expenditure Components



I include the superannuation track, which is endogenously determined in the model, for comparison. The bottom right panel represents total government spending to GDP net of debt servicing costs, and it is this variable that will be important to compare between the models. From 2026 the expenditure to GDP tracks for both models are by and large parallel, which was the goal of the exercise, although the NCGM track is more than a percentage point higher.

With the spending assumptions and tracks in place, I assume that tax rates for the three tax types all adjust to (approximately) balance the budget in each period, which stabilises net debt around the target. Modelling a transition path with variable net debt to GDP provides some challenges. In the absence of government debt, the entire increase in government spending could be covered through the adjustment of a single tax rate as done by Schmitt-Grohe and Uribe (1997), although there is no guarantee this will generate a dynamic equilibrium (see Schmitt-Grohe and Uribe, 1997). It would also be possible to adjust multiple tax rates, where the paths for all tax rates are exogenous except for one, which adjusts to balance the budget, although there is no guarantee that a dynamic equilibrium can be found here either. Both these approaches would be consistent with an exogenous debt track. This might suffice for a deterministic transition path, but I also want the ability to model adverse economic shocks that cause fluctuations in net debt, and I want to allow tax rates to respond to deviations of net debt from the target. To incorporate these features, I introduce a new type of tax rule that includes time-varying target tax rates. In all the ageing population scenarios, I assume a linear transition in the target tax rate from the initial tax rate to the terminal tax rate which turns out to be a reasonable approximation for balancing the budget on the transition path, although it means that government does not exactly match the debt target in all periods. In the absence of a time-varying tax target, the steady state tax target in the tax rule would have to jump from the initial tax rate to the terminal tax rate which means households would be significantly under or over taxed for a period of time, depending on when the jump occurs and the debt target is missed. Fine tuning the exogenous time-varying tax targets or tightening up the response coefficients could help match the debt target more closely on the transition path, although the deviations for the linear tax rate targets are not particularly large.

In addition to some of the exogenous assumptions made, I make a couple of simplifying assumptions regarding net debt to GDP and the gap between interest rates and the growth rate of GDP. These include setting the initial condition for net debt at 48 percent of GDP. This is the peak of the net debt projection over the forecast period and is higher than the current level of net debt to GDP, which is 34 percent. While the projection could be started from a net debt to GDP ratio of 34 percent, the dynamics required to get from 34 percent to 48 percent over the space of two years are quite extreme, so I simplify things by starting the projections from 48 percent, which I also use for the net debt to GDP target in the model.

In many developed countries, New Zealand included, interest rates on government debt are currently below the growth rate of GDP. In some countries, this has been the dominant position over much of history and played an important role in debt consolidations after the Second World War. While financial repression, made possible by regulated financial markets, has almost certainly played a role keeping interest rates low over history, it will likely be more difficult for governments to use this tool going forward (see Eichengreen et al., 2019). Some academic economists are cautiously talking about interest rates being lower than growth as the new normal, driven by secular stagnation, high rates of saving and an ageing population (see Blanchard, 2021, for example). There are a limited number of papers, many using overlapping generations models, that have captured this feature, which requires separating the interest rate paid on government debt from the return on physical capital (see Blanchard, 2019, for example). It is harder to build in a permanently lower interest rate in a representative agent framework, in particular on a transition path. There is currently just one interest rate in the NCGM that is linked to the return on physical capital. Future work could consider how to incorporate a permanently lower interest rate into a representative agent framework, especially if this is the new normal.[31] Even in a framework where real interest rates are larger than GDP growth in the steady state, the initial condition for real interest rates could be set lower than the GDP growth rate. However, this would suggest that interest rates are low because households are more patient, saving more, consuming less and pushing up investment to unrealistic levels. Likewise, reducing the steady state wedge between real interest rates and GDP growth also implies households are more patient, pushing up the steady state investment to GDP ratio to unrealistic levels.

Scenarios analysis that involve shocks (also referred to as stochastic scenarios) are built on top of the baseline projection using the stochastic extended path algorithm. The stochastic extended path algorithm involves looping over a sequence of deterministic perfect foresight problems to trace out a time path for the model variables. At each iteration of the loop, shocks are fed into the first projection period, the deterministic model is solved using Newton's method and the model is rolled forward one period, so that the first projection period from the previous step becomes

[31] One way to do this could be to include imperfect competition in capital markets, putting a wedge between the marginal product of capital and the interest rate paid on debt. Another way could be to put government debt in the utility function. Households would then be willing to hold debt at for a lower rate of return than other savings options because they have a preference for holding debt.

the new initial condition for the next iteration step. This continues for the number of periods that the model is subjected to unanticipated shocks. A more formal treatment of the algorithm can be found in Heer and Maussner (2009) (see also Fair and Taylor, 1983; Boucekkine, 1995; Judd, 1998; Heer and Maussner, 2005). I set the extended path algorithm up so that all the exogenous variables used to construct the baseline scenario remain perfectly anticipated after the intial surprise in the first time period. This means that in the absence of shocks or any other modifications to the assumptions, the extended path algorithm will return the baseline projection. The computational costs of the extended path algorithm are relatively low in comparison with other non-linear solution methods, allowing large models, which might not be solved using other methods, to be solved, or small models to be solved more quickly, and in the absence of uncertainty, accuracy is high.[32] While computationally cheap in comparison with other non-linear solution methods, the extended path algorithm still involves some computational costs which are lowered by using an efficient representation of the algorithm written in Matlab.[33] My code also allows conditional forecasting under exact identification (the number of conditioning/data points to be matched is equal to the number of shocks used to match these points), a feature I make extensive use of when constructing some of the alternative scenarios. A full description of my implementation of the algorithm in Matlab can be found in Binning (2021).

32 In the deterministic case, the method is as accurate as the stopping criteria. In the stochastic case, increased uncertainty combined with increased non-linearity will affect the method's accuracy as certainty equivalence holds.

33 The extended path algorithm is not subject to the curse of dimensionality in the same way that other non-linear solution methods are.

5. Ageing Population Scenarios

In this section I present a baseline ageing population scenario where health and superannuation spending increase, and government raises taxes to try and balance the budget in every period. In addition to the baseline scenario, I look at how constraining some spending affects the results. I also look at the impact of both economic and physical shocks in a number of scenarios built on top of the ageing population baseline. Finally I look at a couple of fiscal consolidation scenarios and some delayed fiscal response scenarios that are also built on top of the baseline ageing population scenario.

5.1 Baseline Ageing Population Scenario

The baseline scenario starts in 2021Q3 when agents (households and firms) are surprised by the entire future paths of population growth, the old-age ratio, the (exogenous) aggregate disutility of working, the exogenous spending tracks, the target tax rates and the terminal steady state. In all subsequent projection periods agents perfectly anticipate the exogenous variables and the terminal steady state. I assume that government credibly commits to keeping the age of eligibility for superannuation at 65 years and older for the next 40 years and to gradually raising health spending from 6 percent of GDP in 2021 to over 10 percent by 2061. Government tries to defend their net debt target by raising taxes to approximately balance the budget in each period, ensuring the economy is always in dynamic equilibrium. The three main average tax rates are raised proportionately so as not to favour one tax type over the others, although the different tax types have different degrees of distortion for the different spending policies, hence alternative combinations could result in lower GDP losses relative to trend over the projection period. The results for the baseline projection are reported in Figures 9 and 10 below. Many of the aggregate variables are reported relative to trend, where trend is the level that would prevail if, starting from their initial levels, macroeconomic aggregates (GDP, the expenditure components and net debt for example) grew at the same rate as total factor productivity and the population over the reporting period.[34]

[34] The NCGM is written in per capita terms. The per capita trend level is the per capita balanced growth path starting from the initial steady state. Per capita macroeconomic aggregates grow at the same rate as the total factor productivity trend. I assume that the aggregate trend level is calculated by multipliying the per capita balanced growth path by the projected population path, so that the percentage deviation from the per capita trend is also the percentage deviation from the aggregate trend. Alternative counterfactual trend assumptions can be made.

Figure 9 — Baseline Projection

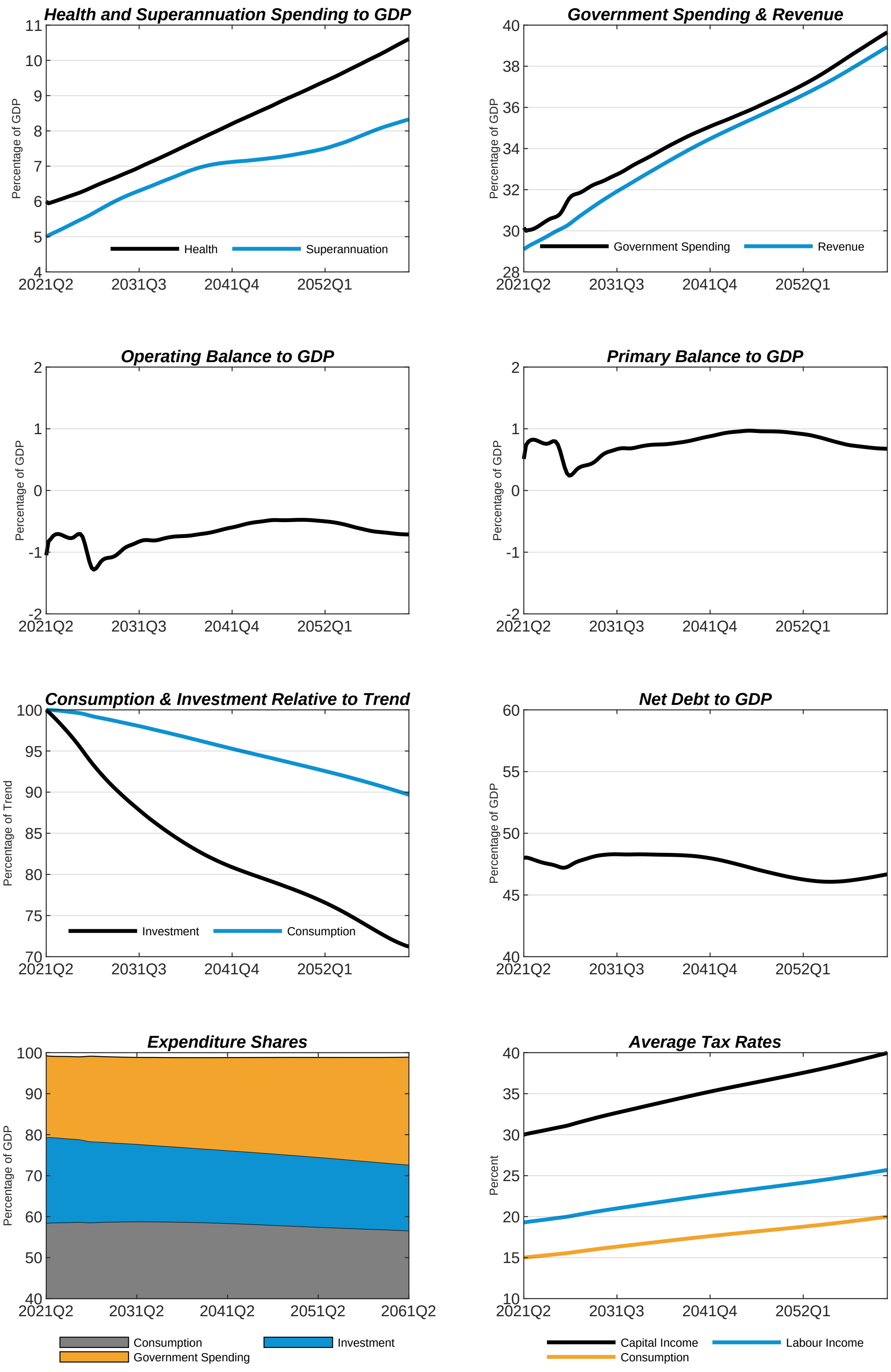
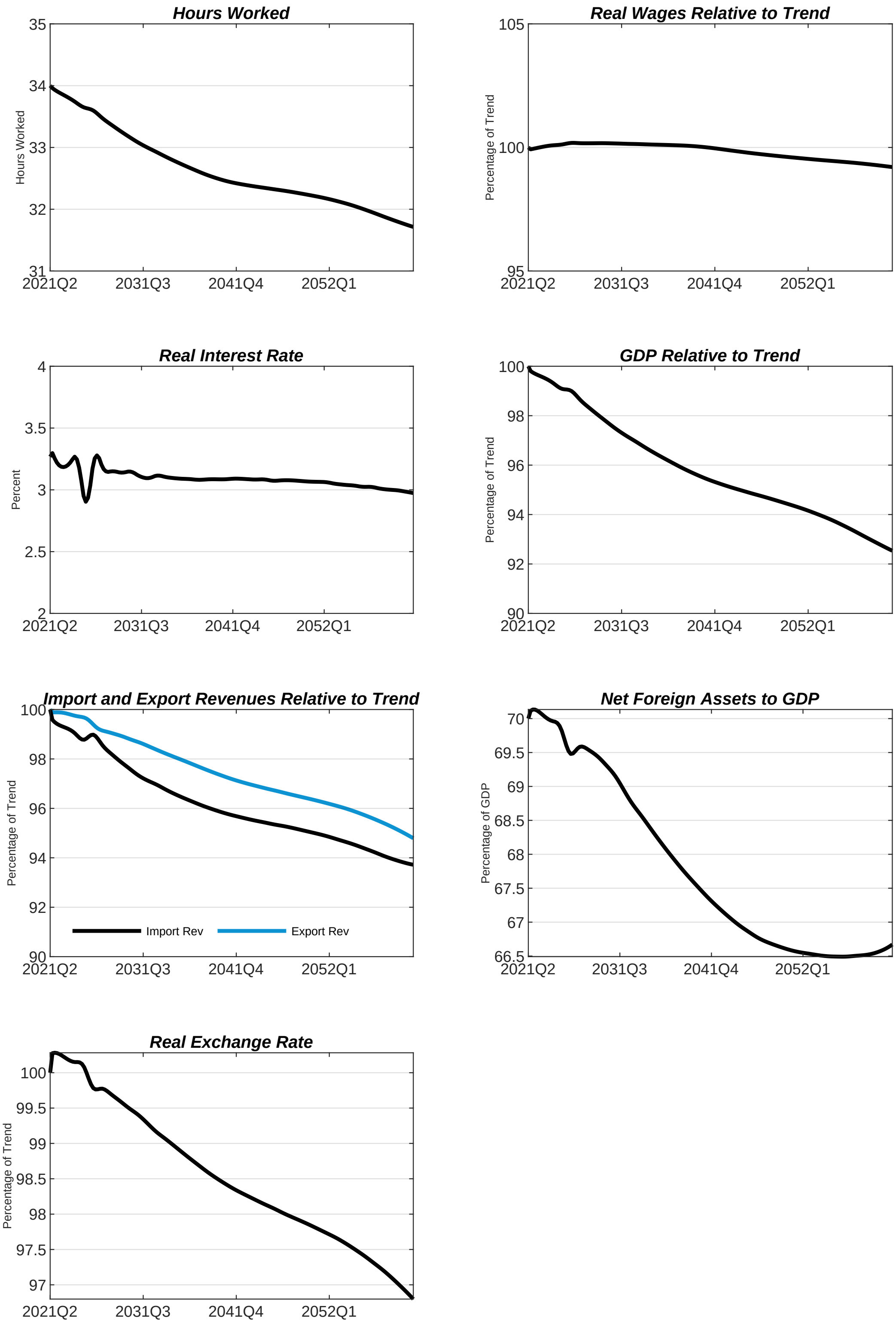


Figure 10 — Baseline Projection



The expected demographic and policy changes work through a number of different channels, including:

- *Declining population growth:* The decline in population growth over the reporting period leads to a reduction in private investment relative to trend. Aggregate investment increases in each period, in part because a larger population requires more plant and machinery to produce more output. Due to the lag between when investment is made and when it becomes productive capital stock, higher expected future population growth means households need to make a small amount of excess investment each period in anticipation of future population needs. Lower population growth means this excess investment can be reduced to maintain equilibrium. The final good that would have been channeled into excess investment at current population growth rates is diverted into consumption as the population growth rate declines. The higher level of consumption makes households feel wealthier, so they reduce their labour supply, all else equal, leading to a reduction in both real GDP and investment relative to trend.

- *Increasing aggregate disutility of working:* The aggregate disutility of working is assumed to increase with time as older cohorts, which assign a higher value to leisure, make up a greater share of the population. This in turn leads to a reduction in the labour supply and hours worked, resulting in lower output, consumption and investment, relative to trend, all else equal.

- *Increasing taxes:* The increase in government spending on health and superannuation is funded through increases in capital and labour income taxes and consumption taxes. Increases in capital taxes reduce the after-tax return on investment. Both the capital stock and the level of investment need to fall to allow the marginal product of capital to increase, which in turn raises the pre-tax return on capital, so that the after-tax return on all assets (physical capital, net government debt and net foreign assets) is equated in equilibrium. A reduction in the capital stock lowers the marginal product of labour, which lowers labour productivity and wages, all else equal, reducing households incentives to work. Higher taxes on labour income reduce the after-tax return from working, reducing labour supply, all else equal. Higher taxes on consumption lower the purchasing power of households and the consumption based rewards from working, reducing the incentives to work, while expectations of future increases in consumption taxes encourage consumers to spend more today and less in the future in a bid to avoid some of the tax burden.

- *Increasing health spending:* Increases in government consumption crowd out private consumption due to the resource constraint.[35] Lower consumption makes households feel less wealthy which raises their willingness to work, usually referred to as the negative wealth effect, which raises the labour supply, all else equal. This partially offsets the negative effects of raising taxes. Flipping this around, the balanced budget permanent government consumption multiplier in the model is zero when consumption taxes finance the increase in government spending, as the distortionary effects of taxation are directly offset by the negative wealth effect under this calibration.[36] The balanced budget permanent government consumption multiplier is negative when labour or capital income taxes finance the increase in government spending as the distortionary effects from taxation are larger than the labour market implications of the negative wealth effect. The balanced budget permanent spending multipliers for this model can be found in Appendix G, along with some discussion about how they compare with similar models in the literature.

The results of the baseline scenario will be a weighting of these different channels, where some responses will offset each other, while others will be reinforcing.

[35] Government purchases leave the private sector with fewer resources, forcing them to reduce their consumption.

[36] I use log utility.

The ratio of health spending to GDP increases from 6 percent in 2021 to over 10 percent in 2061, matching the LTFM track over 2026 to 2061 period. The ratio of superannuation spending to GDP increases, driven by the increase in the proportion of the population over the age of 65. Total primary government spending, as a ratio to GDP, increases from an initial steady state of 28.6 percent to 38.3 percent by the end of the reporting period. Total tax revenue, as a share of GDP, tracks expenses closely, increasing from 29.1 percent of GDP to nearly 39 percent by 2061, as tax rates are adjusted to approximately balance the budget in each period. As a consequence both the ratios of the operating balance and the primary balance to GDP are projected to be reasonably stable over the reporting period, which translates into a stable net debt to GDP path.

Looking at the expenditure components of GDP, investment falls by 28.8 percent relative to trend. This can be broken down into the approximate contribution from the change in the different exogenous variables over the transition path.[37] The reduction in population growth that occurs over the reporting period causes investment to be 10.3 percent lower than trend in 2061. The increase in the aggregate disutility of working accounts for a 3.7 percent reduction in investment. Higher taxes on capital and labour income and consumption expenditure, net of the negative wealth effect, contribute to the remaining 14.9 percent of the fall in investment. Consumption falls by 10.3 percent relative to trend. Higher tax rates and crowding out from increased government expenditure contribute to 9.5 percent of the fall in consumption, while 2.8 percent is due to the increasing aggregate disutility of working caused by an ageing population. Lower population growth contributes to a 2 percent increase in consumption, partially offsetting some of the negative effects of the other exogenous variables. Total non-transfers government spending as a share of GDP increases from 20 percent to 26.3 percent in 2061. Investment as a share of GDP falls from 20.9 percent to 16.1 percent in 2061 due to the reduction in population growth and the increase in capital taxes. Consumption falls from 58.4 percent of GDP to 56.6 percent, due to increases in taxes and crowding out from increased health spending, although this is partially offset by the fall in population growth.

The average tax rate on capital income increases from 30 to 40 percent, the average tax rate on labour income increases from 19 to 26 percent, while the consumption tax rate increases from 15 to 20 percent. The tax to GDP ratio, a proxy for the average tax rate faced by households, increases from 29 to 39 percent, a nearly 10 percentage point increase in average tax rates. I compare my results against those of Attanasio, Kitao, and Violante (2010), who use an overlapping generations model, to investigate increasing health costs and an ageing population in the US. Attanasio, Kitao, and Violante (2010) calculate that labour taxes would need to increase by 14 percentage points between 2005 and 2080 to cover increased pension and health spending obligations, where two thirds of the increase is due to increases in health expenditure. They set capital's share of income to 0.33 which implies that the average tax rate across all sources of income and expenditure faced by households would have to increase by more than 9 percentage points.

[37] The model is non-linear which means the order that exogenous variables are added or removed from the model affects the marginal contribution of that variable to the result. I crudely approximate the contribution of each exogenous variable to the baseline scenario by running a number of counterfactual scenarios. In the first step I run a projection with a given exogenous variable removed, and I calculate the difference between this projection and the baseline projection. In the second step I run a projection where the same exogenous variable is the only exogenous variable added to the model and I calculate the difference between this projection and the initial steady state. I then average these two numbers, repeating the process for all exogenous variables, creating an approximate decomposition of each endogenous model variable into the contributions from each exogenous variable. Due to non-linearities, the decompositions will not necessarily add up to the total change in the projected variable. The marginal contributions from adding the exogenous variables one by one are plotted in Figures 28 - 30 for illustrative purposes, although the ordering that the exogenous variables are added will affect the marginal contributions.

Hours worked fall, mainly due to the aggregate disutility of working increasing over the reporting period, but this is also reinforced by lower population growth and higher taxes on capital and labour income and consumption expenditure, which are required to pay for increased health and superannuation spending. Increased government spending on health services provides a partial offset through the negative wealth channel. Real wages remain reasonably constant over the reporting period, eventually settling 2 percent lower in the terminal steady state. While labour demand falls due to increased taxes on capital income, there is a partial offset due to the increase in the public capital stock.[38] Equilibrium labour falls by nearly as much, due to the increase in the aggregate disutility of working and higher taxes on labour income and consumption expenditure, putting upward pressure on real wages. The real interest rate falls by about 0.25 percentage points over the reporting period due to lower growth in consumption and GDP.

Real GDP declines by about 7.5 percent over the reporting period. Just like investment and consumption, the decline in real GDP can be broken down into the approximate contributions from the different exogenous variables. About 3.1 percent of the decline in GDP comes from the increase in the aggregate disutility of working, which leads to lower labour supply and output. About 0.6 percent of the decline is due to lower population growth, which leads to lower investment, higher consumption and a reduction in the labour supply. And about 3.7 percent is due to the net effects of higher taxes on capital income, labour income and consumption, which lower investment and decrease the labour supply. The overall permanent expenditure multiplier is -0.48, so that every permanent dollar increase in government expenditure leads to a 48 cent permanent fall in GDP. This can be further broken down into a health spending multiplier of -0.32, and a superannuation spending multiplier of -0.7.[39]

Imports fall relative to trend due to lower aggregate demand. In the terminal steady state it is assumed that the ratio of net foreign assets to GDP returns to the same level it started from in the initial steady state. Lower population growth means New Zealand has to run higher trade surpluses relative to GDP to service net foreign debt. The projected fall in GDP relative to trend means net foreign debt needs to be paid down to maintain the same ratio in steady state. This means exports need to fall, but at a slower rate than imports, allowing net exports to settle at a higher level in relative terms. The only way this can happen is if the exchange rate permanently appreciates, which it does over the projection period settling at a lower level in the terminal steady state.[40]

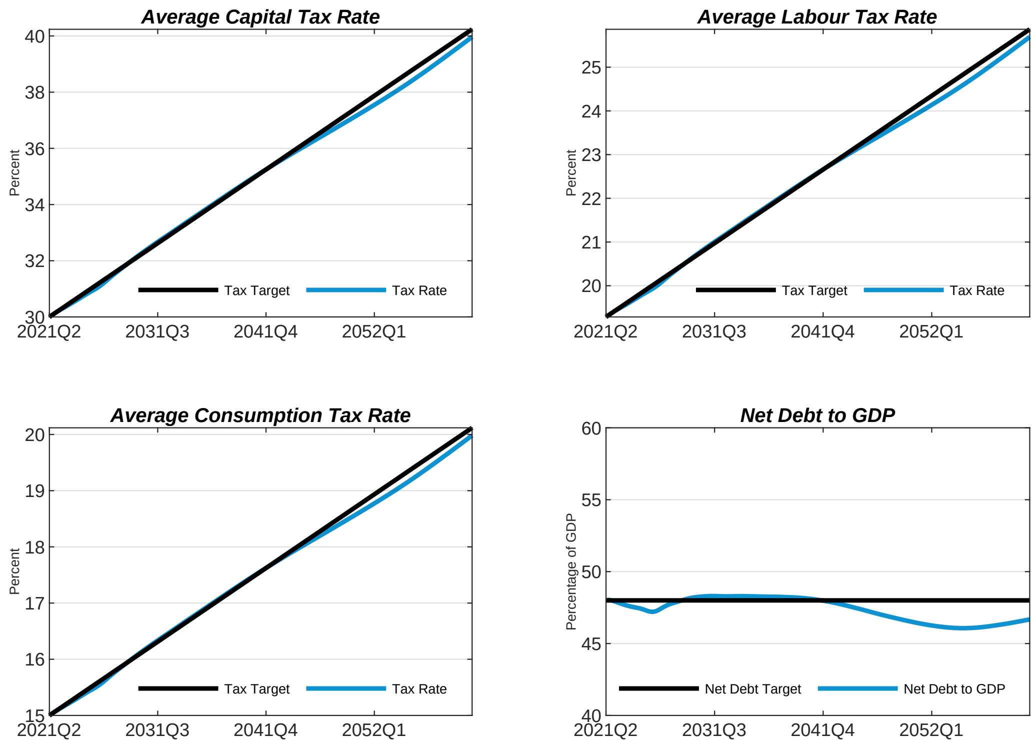
Before moving on to the scenarios, I briefly illustrate the time-varying target tax rates in action. As discussed in Section 4, time-varying target tax rates are introduced as a way of allowing all tax rates to adjust on the transition path while at the same time allowing net debt to GDP to vary in response to shocks. The tax and debt targets are presented in Figure 11 along side the tax rates and the net debt to GDP ratio to illustrate how closely these variables match their targets.

38 Because government investment is set as a ratio to GDP, declines in population growth will increase the public capital stock, which is the opposite of what occurs with the private capital stock.

39 The health spending multiplier is equivalent to the permanent government consumption multiplier and the superannuation spending multiplier is equivalent to the permanent transfers multiplier. The corresponding total expenditure semi-elasticity is -0.41, implying that a 1 percentage point permanent increase in the ratio of government expenditure to GDP results in a 0.41 percent permanent reduction in the level of GDP. This is in the neighbourhood of the -0.5 semi-elasticity calculated using empirical estimates for OECD countries in Gemmell, Kneller, and Sanz (2011). The corresponding health semi-elasticity in the NCGM is -0.31 and the superannuation semi-elasticity is -0.66. The multipliers and semi-elasticities in the NCGM are a function of the different tax types used to balance the budget in the long run. Different combinations of taxation will result in different multipliers and semi-elasticities. The permanent balanced budget multipliers for different policies in the model are reported by both spending type and tax type in Appendix G.

40 A downward movement in the exchange rate is an appreciation, while an upward movement is a depreciation.

Figure 11 — Tax and Net Debt Targets



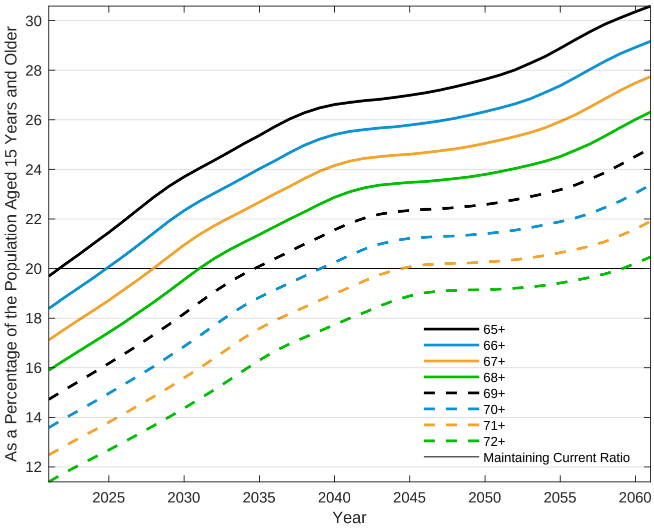
The tax targets are assumed to follow a linear path from the initial steady state to the terminal steady state. At the terminal steady state the tax targets are set equal to the tax rates required to balance the budget for the chosen levels of spending and debt in the terminal steady state. The tax rates and net debt follow their respective targets quite closely up until 2040. After 2040, the target tax rates are set a little too high in relation to spending increases, resulting in the tax rates being set lower than their respective targets. However, they are not set low enough leading net debt to GDP to undershoot its target. This occurs because the increase in the share of the population over the age of 65 is not linear, noticeably slowing down after 2040, before experiencing a second wind towards the end of the reporting period. The tax targets could be fine tuned further to reduce the tax and debt gaps after 2040, but I do not do this here, because the gaps are still relatively small.

5.2 Spending Restraint

To understand the marginal contributions of increased health expenditure and increased superannuation spending, I consider two alternative spending policies in this section. Under the first policy, health spending grows at the same rate as GDP over the reporting horizon, so that the ratio remains constant at its initial value. Under the second policy, the proportion of the population eligible for superannuation is kept constant at the current level. The policy that keeps health at a constant share of GDP is interchangeable with any other policy that would keep government consumption at a constant share over the reporting period. The superannuation policy is interchangeable with any policy that would see transfers spending remain at a (reasonably) constant share of GDP over the reporting period. Keeping the proportion of the population eligible for superannuation constant at current levels would be equivalent to gradually increasing the age of eligibility to 72 by 2061 as illustrated in Figure 12.[41]

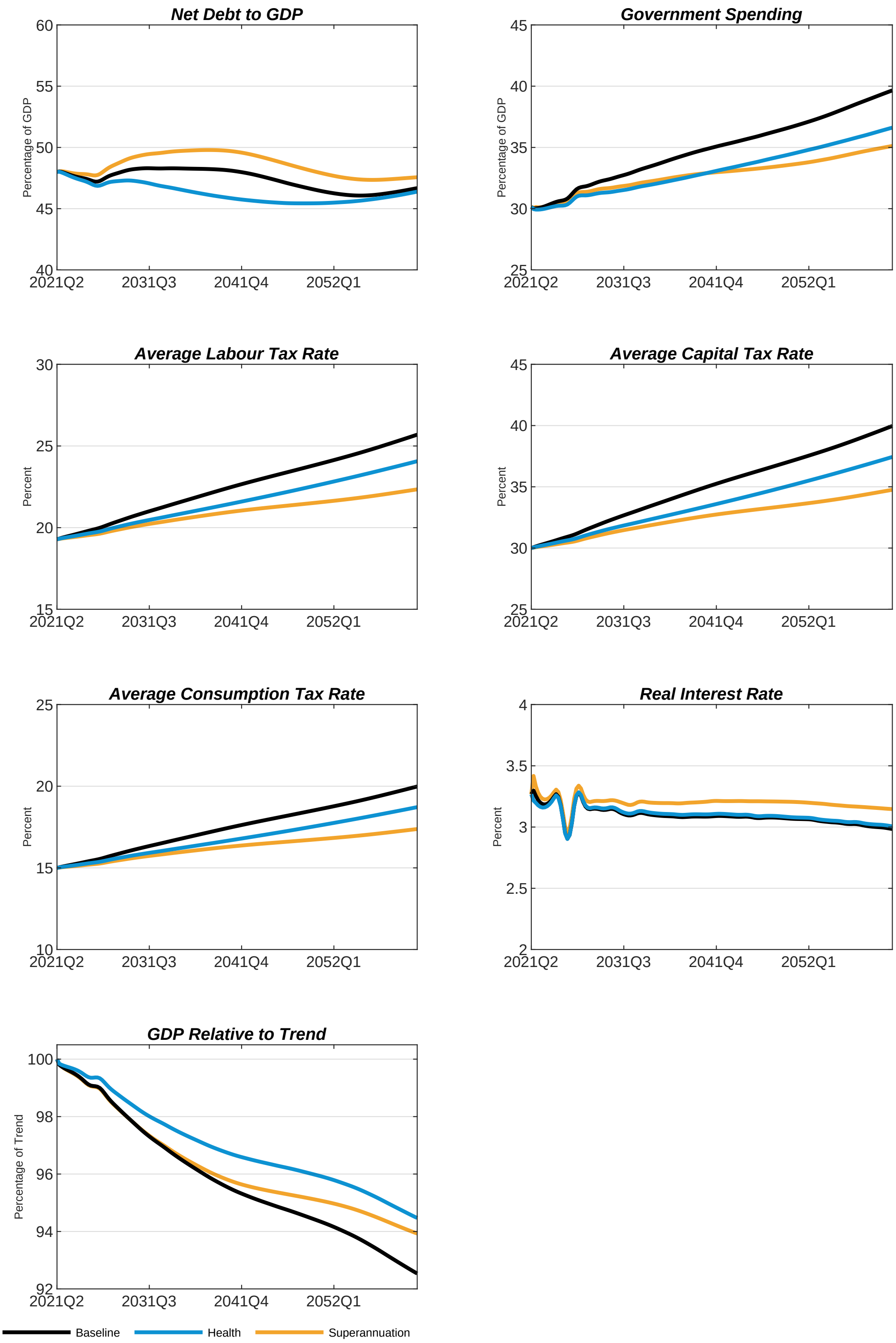
[41] Based on Statistics NZ projections the age of eligibility would need to increase to 75 in 2073 if the proportion of the population eligible were to be kept constant at the current level.

Figure 12 — Old-Age Ratios



A subset of the projection results are presented in Figure 13 below.

Figure 13 — Spending Reduction



The ratio of total primary government spending to GDP for both alternative spending policies look similar until about 2040, where there is a noticeable slow down in the increase in superannuation spending as a share of GDP, causing the paths to diverge. This is driven by the slow down in the growth of the population aged over 65 relative to the working age population. Health spending, by comparison, is projected to grow at a reasonably constant rate in excess of GDP over the entire reporting period. In the constant health spending scenario, total government spending increases to 33.6 percent of GDP by 2061, compared with 38.3 percent in the baseline and 35.2 percent under the constant superannuation policy. Reflecting the relative increases in total government spending; capital, labour and consumption taxes are lower than the baseline when superannuation spending is kept at a reasonably constant share of GDP, and lower again when it is health spending to GDP that is kept constant. While the constant superannuation policy is more costly than the constant health policy from a tax and spending perspective, due to health spending increasing by more than superannuation spending, the fall in GDP relative to trend is larger in the constant health spending scenario. This is because increases in transfers payments only affect agents' behaviour through the tax channel in this model. By contrast increases in health spending (government consumption) crowd out some private consumption and cause households to work more due to negative wealth effects. The increase in the labour supply provides a partial offset to the negative effects of higher taxes, resulting in a smaller GDP loss, compared with the scenario where transfers payments increase.

5.3 Recessions

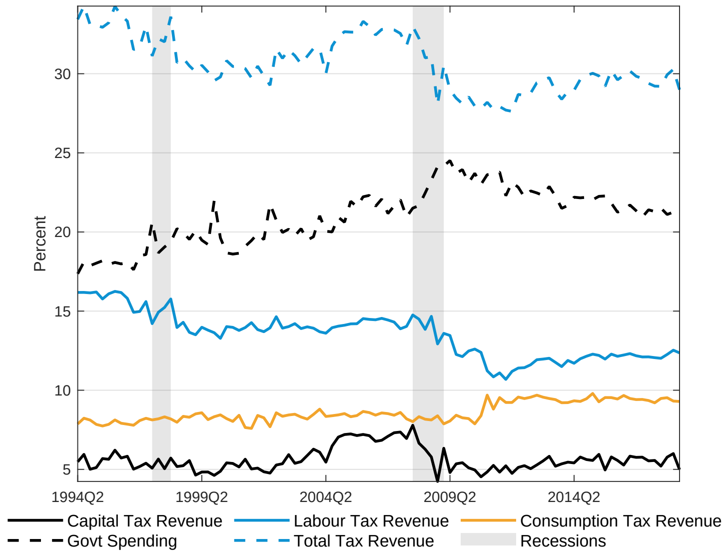
Recessions usually add to fiscal pressures. Governments typically increase spending to stimulate economic activity in efforts to smooth the business cycle. Tax revenues usually fall due to lower incomes, declining sales revenues and falling consumption. In some instances governments may cut taxes during recessions, attempting to further stimulate the economy. At the same time rising unemployment leads to higher benefits and transfers payments. When automatic stabilisers and counter-cyclical fiscal policy are allowed to operate, government debt to GDP more often than not expands during recessions. To better understand how a recession, or sequence of recessions could affect the Crown's fiscal position over the next 40 years, I construct scenarios with a single recession and a series of recessions on top of the baseline ageing population scenario.

To calibrate the size and duration of the recession(s) I look at the average size of recessions in New Zealand over history. Hall and McDermott (2016) provide a summary of the business cycle properties of the New Zealand economy since the Second World War. Using the Bry and Boschan algorithm (Bry and Boschan, 1971) they identify nine recessions, where the average duration of the contraction period (from peak to trough) is 4.2 quarters, the average duration of the expansion period (trough to peak) is 30 quarters, the average amplitude (from peak to trough) is -4 percent, and the average cumulative GDP loss is -9.9 percent (calculated using the triangle method from Pagan, 2005). These numbers are used as a rough guide for calibrating the scenario. I also compare a subset of projections from the model against their counterparts in the data for the three most recent recessions where data is available.

To get a rough indication of how fiscal policy and the automatic stabilisers have operated during recessions I look at how tax revenue and government spending, as a fraction of GDP, have fluctuated over recent history. I use tax receipts data which start in 1993, so that only the Asian crisis and the GFC are covered.[42] Figure 14 shows how total tax revenue, capital income tax revenue, labour income tax revenue, consumption tax revenue and total government spending as a share of GDP have varied over history. I also highlight the recessions over this period as dated by Hall and McDermott (2016).

[42] Data definitions can be found in Appendix H.

Figure 14 — Tax Revenue and Government Spending to GDP



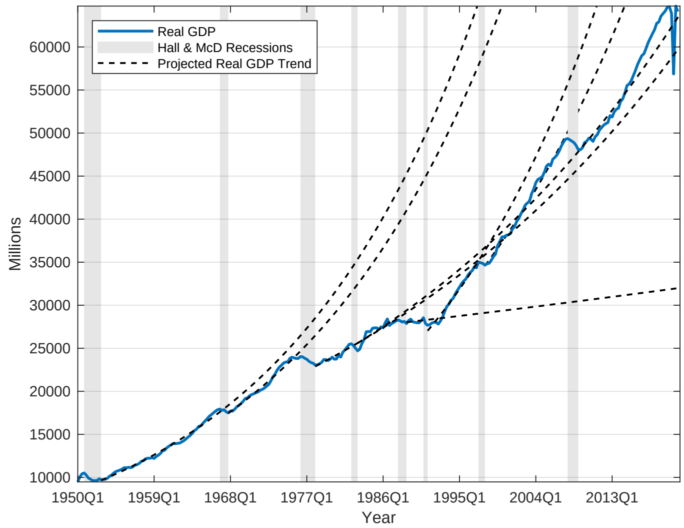
Most of the action occurs before and after the global financial crisis (GFC). Some of this is due to the automatic stabilisers operating. Some of this is due to cuts in personal and business tax rates, and increases in the GST rate that occurred between 2009 and 2011. The ratio of total tax revenue to GDP fell from a peak of 33 percent before the GFC, to a trough of 28 percent. Labour tax revenue as a share of GDP fell from a peak of nearly 15 percent before the GFC to a trough of 11 percent. Capital income tax revenue as a share of GDP fell from a peak of 8 percent before the GFC to 5 percent after the GFC. Consumption tax was reasonably flat until 2010 when GST rate was increased. On the expenditure side, total non-transfers government spending increased from 21 percent of GDP before the GFC to just under 25 percent of GDP after the GFC.

It is quite common for recessions to have what look like permanent effects on the levels of key macroeconomic aggregates, like GDP and the expenditure components, which can have implications for how recessions should be modelled (see Cerra and Saxena, 2003; Cerra and Saxena, 2008; Cerra and Saxena, 2017, for a discussion).[43] I investigate whether there has been a permanent component to historical recessions in New Zealand by using the Hall and McDermott (2016) production GDP series and business cycle dates to construct counterfactual pre-recession trend paths. This is done by fitting a log time trend over each expansion period and projecting this forward in time to the end of history. These paths represent counterfactual trajectories that the economy could have obtained in the absence of each recession. I use these counterfactual paths to determine whether GDP returned to its pre-recession trend level once the economy had recovered.[44] The results from this simplistic analysis are presented in Figure 15 below.

[43] See Eo and Morley (2017) for an alternative explanation of the GFC and the sluggish GDP response in the US.

[44] This assumption depends on the interpretation of trend and data. Another interpretation could be that the economy grew faster than trend during the boom and that the trend should be calculated over the entire business cycle, or even using multiple business cycles.

Figure 15 — Real GDP



Under these assumptions it appears the 1967, 1976, 1997 (Asian crisis) and the 2007 (the GFC) recessions all display a permanent reduction in trend GDP, with GDP failing to return to the pre-recession trend lines following each of these recessions. The 1982, 1988 and 1991 recessions are consistent with the classic view that GDP returns to or exceeds trend once it has recovered, although these recessions all occurred during a period of weak GDP growth. Because half of the recessions seem to have permanent effects, and these permanent effects help generate more plausible net debt to GDP responses in the model, I include a permanent component for each of the recessions that I model.

Econometric evidence from structural models typically indicates that recessions are the result of multiple shocks simultaneously hitting the economy.[45] No two recessions are alike and practitioners are yet to locate the elusive recession shock in these models. Constructing plausible recession scenarios requires a cocktail of shocks. I produce the recessions scenarios using the following key modelling assumptions and judgements:

- The recessions are constructed with a contraction phase of 4 quarters where GDP falls (peak to trough) by 4 percent. The business cycle length is set to 32 quarters (8 years is a rounder number than the Hall and McDermott (2016) duration of 30 quarters). The responses of consumption, investment, government spending, exports, imports, hours worked per person, the real interest rate, transfers payments, tax revenue and net debt to GDP, are chosen to fall within plausible ranges based on the available data for these variables and the Hall and McDermott (2016) recession dates.
- I temporarily lower the target tax rates for labour and capital income tax rates and the consumption tax rate by roughly 1.5 percentage points during each recession. At the same time I introduced a temporary but persistent decline in the fiscal authority's willingness to stabilise net debt over the recession period. Together these combine to produce about a 2 percentage point reduction in tax revenue to GDP, over the recession period. This is within the range of what was observed during the GFC. Debt consolidation after the recession requires a reasonably strong tax response to pay down debt.

45 This may be due to a wider problem in structural models where estimated shocks are cross-correlated where they should be uncorrelated as a condition of their identification. Liu, Pagan, and Robinson (2018) attribute this to the models being overidentified, so that moment conditions on the shocks are not be satisfied, while Andrle, Bruha, and Solmaz (2016) put this down to model misspecification and the absence of a true business cycle shock.

- I add a sequence of health spending shocks to ensure that health spending follows a path close to what it would have followed in the absence of a recession. This is a consequence of setting health spending as a share of GDP, rather than as a level. Because GDP falls during the recession, health spending will rise as a share of GDP.

- I add a sequence of general government consumption and government investment shocks so that the ratio of total non-transfers government spending to GDP is 1 percentage point higher than what it would have been in the absence of a recession. I add transfers shocks to ensure that the ratio of transfers payments to GDP are 0.75 percentage points higher than what they would have been in the absence of a recession.

- I add a sequence of risk-premium shocks to prevent real interest rates falling too much. In the absence of these shocks real interest rates fall by more than what would be considered plausible. While real interest rates can and do go negative, the effective lower bound on nominal interest rates should have some impact on the range of real interest rates, and by ignoring monetary policy the model is not able to account for the effective lower bound. With these shocks added the real interest rate movement falls within the range of recent recessions.

- I add a sequence of consumption preference and investment specific technology shocks that see consumption fall by nearly 4 percent and investment fall by nearly 18 percent from peak to trough.

- I add a sequence of export demand shocks that results in an export track that is 2 percent lower than the peak but recovers quite quickly.

- I add a sequence of negative labour demand and supply shocks to lower hours worked and real wages.

- To engineer a permanent shift in trend GDP, I need to generate a permanent shift in trend total factor productivity.[46] To capture a 4 percent, peak to trough, decline in GDP over 4 quarters, which is the stylised fact for post World War II recessions in New Zealand, I assume that total factor productivity falls 0.75 percent per quarter for 4 quarters.[47]

The results from the recessions scenario are presented in Figure 16 and the relative increase in net debt from each recession are presented in Figure 17.

46 There are two trends in the model, total factor productivity and population.

47 There is some debate in the literature about whether the permanent shift in potential GDP is due to supply or demand shocks (hysteresis). Cerra, Fatás, and Saxena (2020) discuss how demand shocks can have permanent effects on total factor productivity when growth is endogenous, a channel missing from this model. In empirical work, Furlanetto et al. (2021) find that permanent demand (hysteresis) shocks played a large role in the GFC in the US, while Benati and Lubik (2021) find no evidence of hysteresis in US data.

Figure 16 — Recession Scenarios

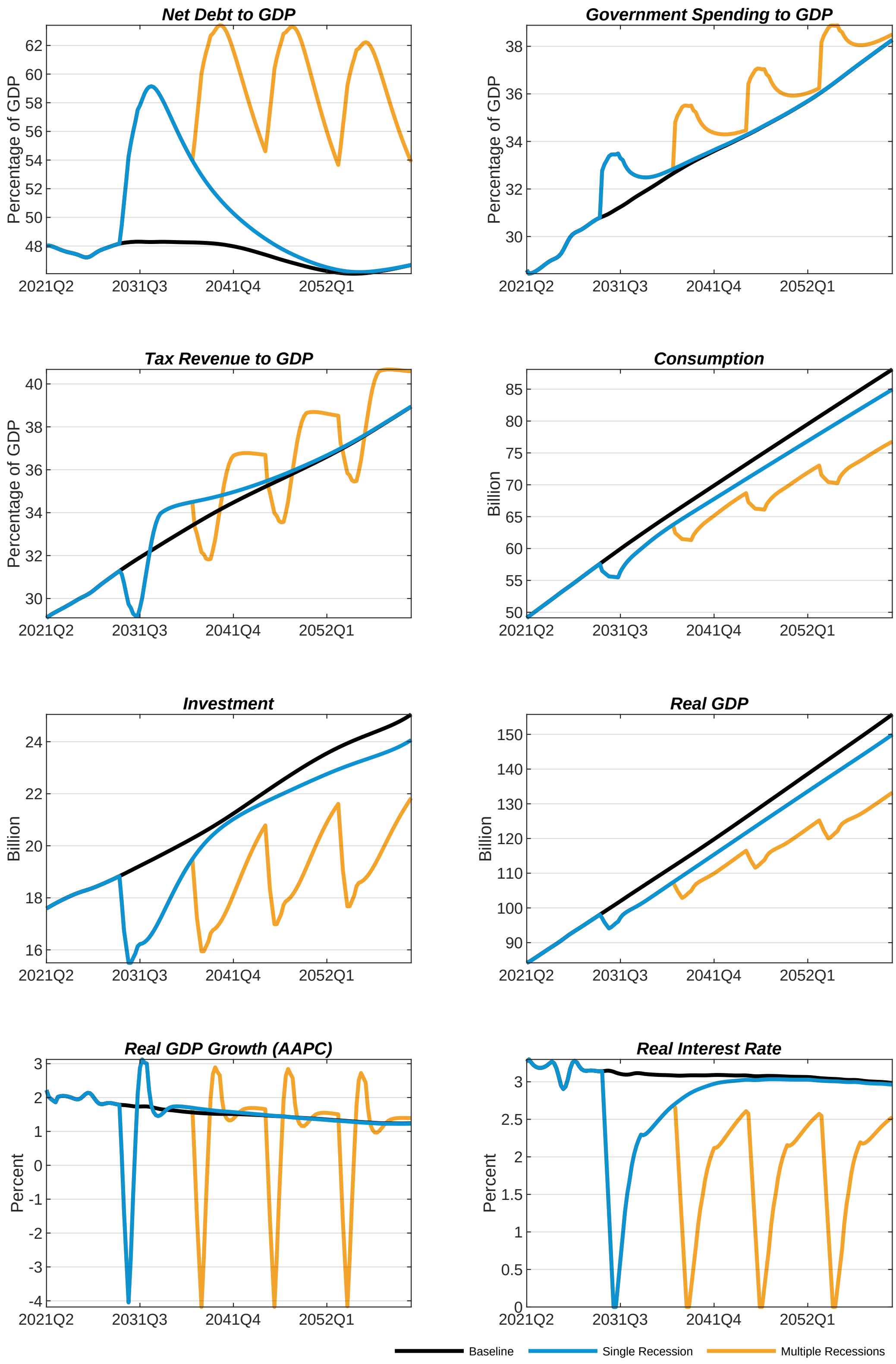
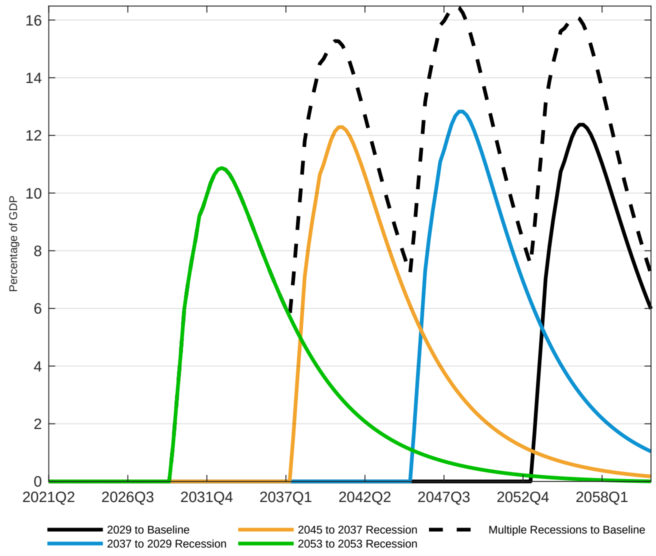


Figure 17 — Relative Increase in Net Debt to GDP



Recessions add to net government debt. For the calibrated recessions scenario, each recession adds between 11 and nearly 13 percentage points to pre-recession net debt to GDP levels and raises net debt to GDP by up to 16 percentage points relative to the no recession baseline. Total primary government spending increases by more than 2 percent as a share of GDP, reflecting the fall in GDP and the increase in government consumption, investment and transfers payments during each recession, before converging back to the baseline spending track. Tax revenue as a share of GDP falls by a little over 2 percent for each recession. As the recession comes to an end, government needs to raise taxes reasonably aggressively to start the debt consolidation process. Following each recession, the average tax rate needs to remain higher than the baseline in order to stabilise and pay down debt. Successive recessions mean that consumption, investment and GDP never return to their baseline ageing population tracks. This is in part due to the hysteresis-type assumption made in the modelling, but it is also because the baseline is built around the average expected growth rate for total factor productivity, which takes into account boom and bust periods. When recessions are factored in, the average total factor productivity growth rate is much lower in the recessions scenario than it is in the baseline. To remedy this, the total factor productivity growth rate in normal times should be higher in the recessions scenario, such that the total factor producivity tracks in both the baseline scenario and the recessions scenario start at the same starting point, and finish at the same end point. Under such an assumption the average total factor productivity growth would then be the same in both scenarios. To avoid additional complications, I ignore this fact for these simulations.[48] Each recession leads to year on year GDP growth falling to negative 4 percent, before rising again. The real interest rate falls after each recession. When there is a sequence of recessions, the interest rate never returns to its baseline level.

The GDP projections from the model are plotted against seven of the most recent recessions, using the Hall and McDermott (2016) GDP series. Some of the other model variables are plotted against the three most recent recessions, where data is available, in Figures 18 and 19.

48 This also means that I am not able to match the Hall and McDermott (2016) stylized facts for expansions.

Figure 18 — Recessions

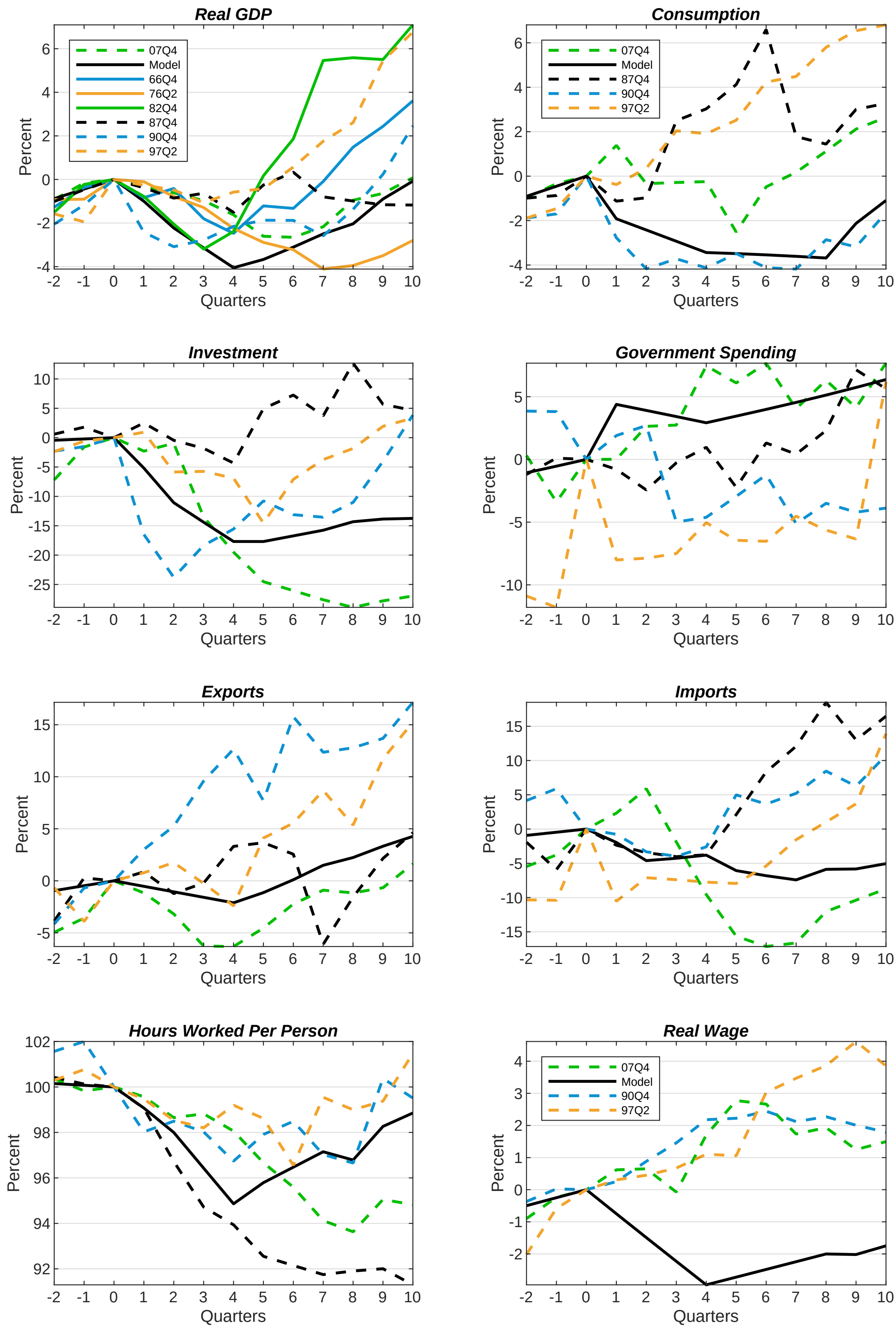
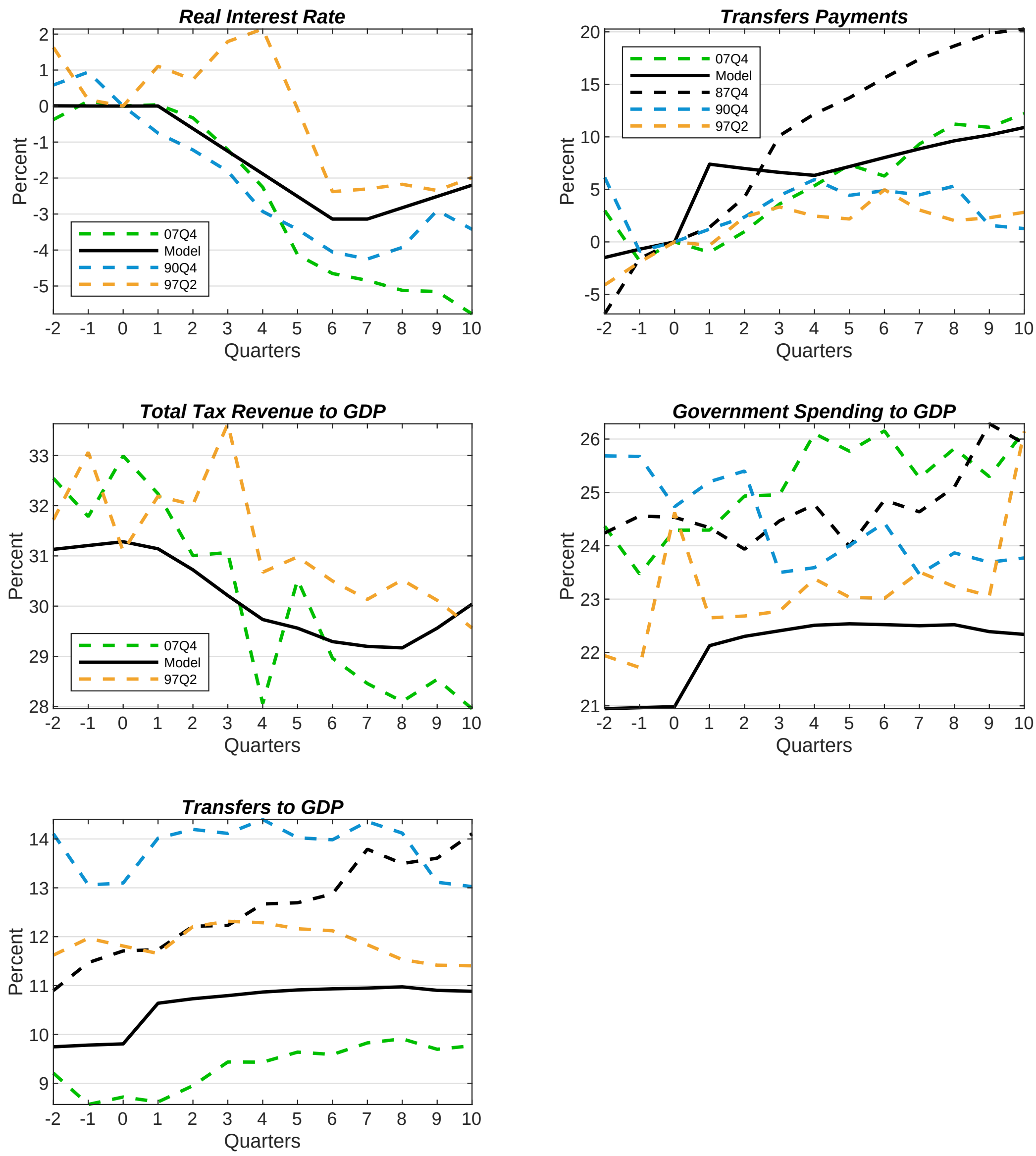


Figure 19 — Recessions



I compare the GDP track from the single recession scenario, which should be representative of each of the recessions modelled in the multiple recessions scenario, with actual GDP data for post World War II recessions by looking at the percentage difference from the peak before the recession, through to 10 quarters following the recession. Due to data restrictions, I compare the relative movements in the GDP components and a number of other model variables in the single recession scenario against actual data for the 1991 recession, the Asian crisis and the global financial crisis. The amplitude of the fall in GDP in the model is about the same as the 1976 recession, although it is shorter in duration, and a bit larger than the fall in GDP that occured in the GFC and the 1991 recessions. This is because the most recent recessions in the post World War II sample are smaller than the recessions that occured immediately after the war. The consumption profile in the scenarios is very similar to the 1991 recession, matching both the amplitude and duration. The investment profile from the model scenarios initially lies in between the 1991 recession and the GFC before recovering, while investment was markedly lower for longer during and after the GFC. Government spending (consumption and investment) spikes a bit higher in the model scenarios, than observed in recent recessions, before settling below the post GFC track. Exports and imports from the scenarios sit within the range of previous recessions, as do hours worked, while real wages are lower in the scenario than they are in the most recent recessions. The remaining variables; real interest rates, transfers payments, tax revenue to GDP, government spending to GDP and transfers spending to GDP all sit within the range or the range of variation in the case of government spending to GDP, observed in the most recent recessions.

5.4 An Earthquake

New Zealand sits at the intersection of the Australian and Pacific tectonic plates, which forms a section of "the Ring of Fire", making it especially prone to physical shocks like earthquakes and volcanic eruptions. Recent experience with the Canterbury earthquakes is a reminder that extreme events can and do happen, and that as well as being costly in human terms, with loss of life, injury and the ongoing impacts on survivors' mental health, they are also expensive in terms of rebuilding costs and lost economic activity. While the EQC natural disaster fund provided some coverage for the Canterbury earthquakes, it has since been largely depleted, meaning direct government financing will need to play a much larger role if a major natural disaster were to occur in the near future. In this section, I model an earthquake scenario as an important way to illustrate and assess the fiscal costs of a large natural disaster.

I have chosen to model a Wellington based earthquake that is 50 percent more destructive than the 2010 - 2011 Canterbury earthquakes. A 1 in 1000 year event in Wellington is typically used by both the insurance industry and geological modellers as a benchmark for calculating earthquake liabilities for a major earthquake event (see Barksby, 2021; Dean, 2011; Cousins, King, and Kanga, 2012, for example). By contrast the 2010 - 2011 Canterbury earthquakes have been assessed to be significantly stronger than a 1 in 1000 year event making an earthquake that is 50 percent more destructive even less likely in that region (see Dean, 2011; Kaiser et al., 2012, for example). And while the Auckland CBD is located between 25 and 40km from known active faults capable of generating earthquakes of magnitudes between 6.7 to 7.2, the chances of these occurring are much more remote with these fault lines expected to rupture once every 10,000 to 20,000 years, on average (see Cousins, Deligne, and Nayyerloo, 2014). Furthermore, GNS modelling suggests that if a 1 in 5000 year event were to hit the Auckland region it would cause $1.8 billion in repair costs, accounting for 0.6 percent of the regions total building value, falling well below the level of destruction that I model (see Cousins, Deligne, and Nayyerloo, 2014).

Following the Canterbury earthquakes about 2 percent of the nation's physical capital stock was written off.[49] I model an earthquake or series of earthquakes in the Wellington region that are 50 percent more destructive than the Canterbury earthquakes, destroying 3 percent of the public and private capital stocks. This is supported by GNS modelling which suggests that a sequence of larger earthquakes in the Wellington region could be 50 percent more destructive than the sequence of earthquakes that hit Canterbury in 2010 and 2011 (see Cousins, King, and Kanga, 2012). While modelling an even larger scenario would be a greater fiscal test, it would also be less plausible and potentially introduce a number of non-linearities in the fiscal responses. In particular, a much larger event could mean that insurance companies are no longer able to meet all their commitments and government may be called on to cover a greater share of private costs. In the absence of any better data or judgement and to simplify the analysis, I use the Canterbury earthquakes as my reference point for determinining what the fiscal policy response might look like for a large earthquake or sequence of earthquakes in the Wellington region. However, Wellington differs from Canterbury in many ways, so there is no guarantee that a proportionate fiscal response based on the Canterbury earthquakes, scaled to match a large earthquake in Wellington, would be the appropriate response to a Wellington based earthquake.

[49] Statistics New Zealand believe that the value of the capital stock destroyed or written off in the Canterbury earthquakes was $14.8 billion in current prices. The productive capital stock at the time was just over $700 billion.

Furthermore, there has been no analysis investigating whether the government's response to the Canterbury earthquakes was optimal or appropriate.

The key modelling assumptions I make to construct the scenario are listed below. All spending assumptions are reported relative to steady state output because the model is written in real terms and the earthquake occurs in the future making it more difficult to factor in an appropriate amount of inflation:

- The earthquake occurs in 2028Q2, sufficiently after the COVID shock that the scenario is not affected too much by COVID conditions and sufficiently early on in the projection period to allow enough time to model the medium and longer run dynamics of the shock.
- The initial economic effects of the earthquake are constructed by simultaneously hitting the economy with a capital destruction shock, that destroys 3 percent of the public and private capital stocks and a permanent total factor productivity shock, which lowers the total factor productivity trend by 3 percent. Using a permanent total factor productivity shock is consistent with the disaster risk literature (see Barro, 2006; Gourio, 2012, for example). It is also consistent with a particular view of recessions where GDP does not return to its original trend path.[50]
- Following the recession private investment ramps up by 2.04 percent of steady state real GDP after 3 years, remaining at this level for 7 years, before tapering off. I obtain 2.04 percent by using the private sector response to the Canterbury earthquakes, and increasing this by 50 percent for the Wellington earthquake. More explicitly, I make the following calculations,

$$\begin{aligned}\frac{\mathcal{I}_x}{\mathcal{Y}} &= 100 \times (1+x) \times \frac{\frac{1}{n}\sum_{k=0}^{n-1}\mathcal{I}_{t+k}}{\frac{1}{n}\sum_{k=0}^{n-1}\mathcal{Y}_{t+k}}, \\ &= 100 \times 1.5 \times \frac{\frac{1}{10}\times 34}{250}, \\ &= 2.04,\end{aligned}$$

- where:
  - $\frac{\mathcal{I}_x}{\mathcal{Y}}$: The average increase in private investment due to the Wellington earthquake.
  - x : The percentage increase/decrease over the average Canterbury earthquake spend.
  - $\mathcal{I}_{t+k}$: Nominal investment in period $t+k$.
  - $\mathcal{Y}_{t+k}$: Nominal GDP in period $t+k$.
  - n : Number of years.
- Total private investment over the 10 year period ($\sum_{k=0}^{n-1}\mathcal{I}_{t+k}$) is about \$34 billion, which is the sum of \$11.8 billion paid out by the EQC (see EQC, 2020) and \$22.27 billion in private insurance payouts (see ICNZ, 2021).
- Nominal GDP for New Zealand in 2015 was about \$250 billion which I use as my average GDP over the period ($\frac{1}{n}\sum_{k=0}^{n-1}\mathcal{Y}_{t+k}$).
- I assume the spending period is 10 years and use 1.5 to scale the numbers for an earthquake that is 50 percent more destructive than the Canterbury earthquakes.

50 The only way to engineer permanent shifts in trends in models like these is through a permanent technology shock. The evidence is mixed on the GDP impacts of natural disasters, but to make this an interesting stress/resilience test, and to get larger movements in net debt to GDP without going overboard on the tax or spending responses, GDP needs to fall, and it helps if the fall is persistent or permanent.

- Government investment after the earthquake ramps up to 0.42 percent of steady state real GDP after 3 years, remaining at this level for 7 years, before tapering off. I obtain 0.42 from the expression,

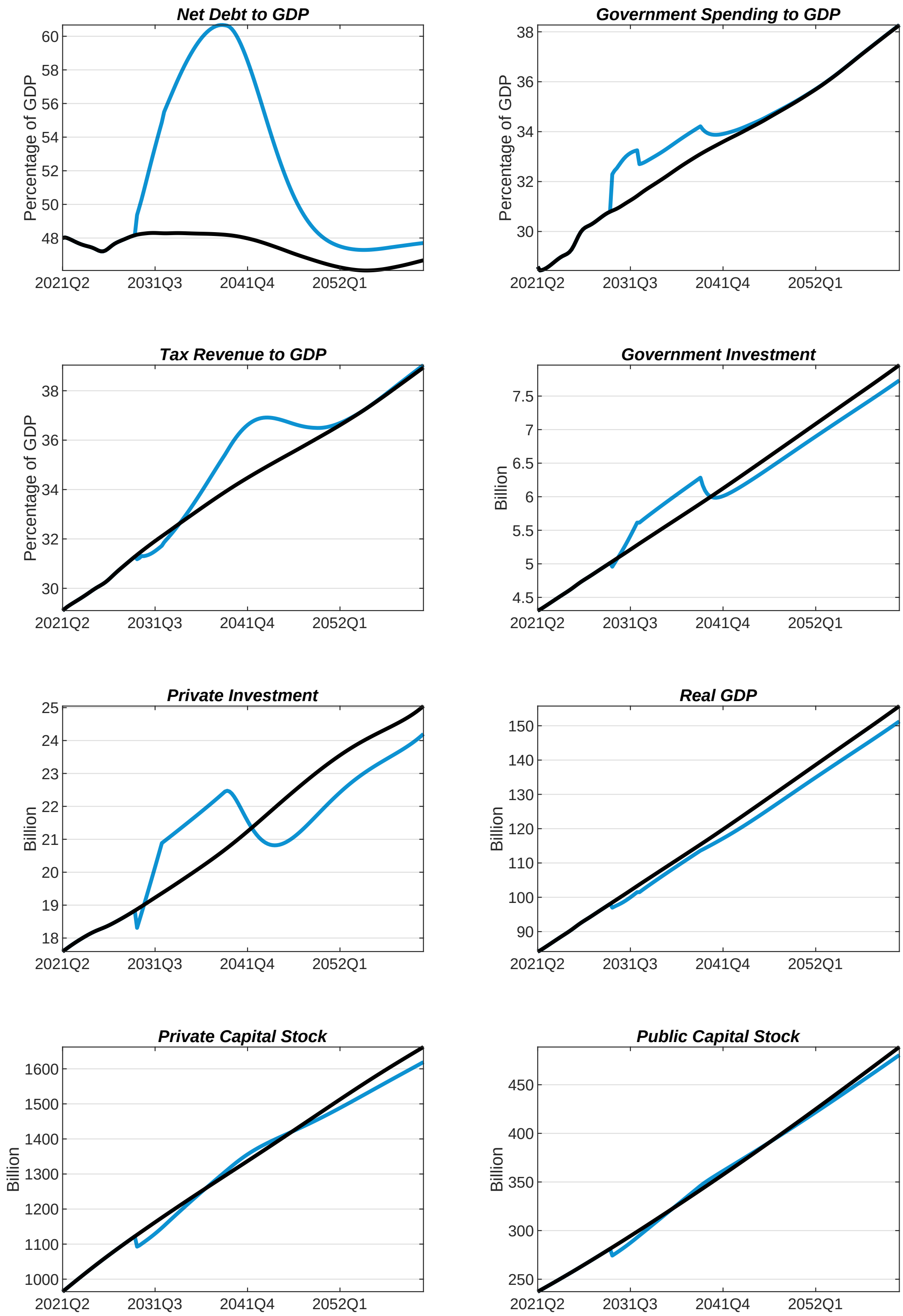
$$\begin{aligned}\frac{\mathcal{G}_{I,x}}{\mathcal{Y}} &= 100 \times (1+x) \times \frac{\frac{1}{n}\sum_{k=0}^{n-1} \mathcal{G}_{I,t+k}}{\frac{1}{n}\sum_{k=0}^{n-1} \mathcal{Y}_{t+k}}, \\ &= 100 \times 1.5 \times \frac{\frac{1}{10} \times 7}{250}, \\ &= 0.42,\end{aligned}$$

- where $\frac{\mathcal{G}_{I,x}}{\mathcal{Y}}$ is the average increase in government investment following the Wellington earthquake and $\mathcal{G}_{I,t+k}$ is nominal government investment in period $t+k$.
- According to Wood, Noy, and Parker (2016), the total public infrastructure spend up to 2015 was about $7 billion and was considered largely complete at that time. I assume that all infrastructure spending was government investment and set $\sum_{k=0}^{n-1} \mathcal{G}_{I,t+k} = 7$.
- This means the total additional spend due to the Canterbury earthquake was about $\sum_{k=0}^{n-1} \mathcal{I}_{t+k} + \mathcal{G}_{I,t+k} = 34 + 7 = 41$, or $41 billion.
- On top of this I add government consumption and transfers spending shocks to raise each by an amount equal to 1 percent of steady state GDP for 3 years, before tapering off. This is to try and match a spending semi-elasticity of 1.15 (a 1 percent decrease in GDP would result in a 1.15 percentage point increase in the expenditure/GDP ratio).[51]
- I lower the target tax rates by a small amount for a year, before allowing them to return to their long-run levels. I temporarily relax the government's tax response to deviations of debt from its target, which allows net debt to increase. It then takes a while for tax rates to adjust and return net debt to target. I aim for a smaller initial decrease in tax revenues, to match a tax semi-elasticity of 0.15 (a 1 percent fall in GDP should lower the revenue to GDP ratio by 0.15 percentage points).[52] In the medium to long run, tax rates need to climb faster than in the baseline scenario to ensure the rebuild is paid for, on top of increasing health and superannuation costs, and returning net debt to target.
- The real interest rate is lowered by 50 basis points for two years, before it is allowed to endogenously increase. The RBNZ cut interest rates by 50 basis points following the Canterbury earthquakes, and I assume for simplicity's sake that there is a close mapping between the nominal and real interest rates in this exercise.[53]

The results from the earthquake scenario are reported in Figure 20.

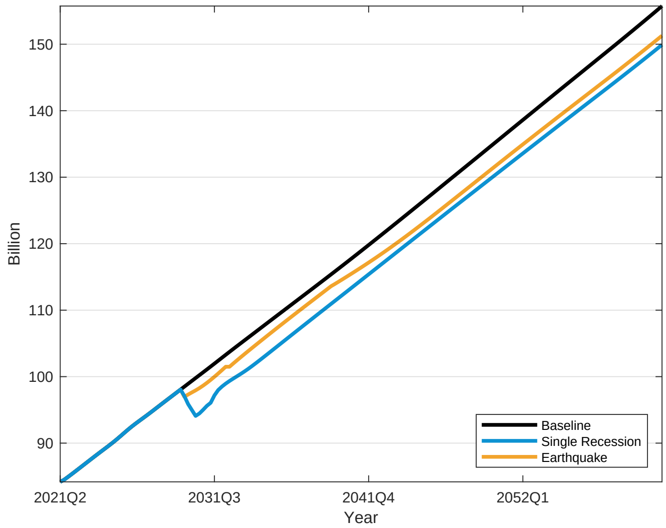
[51] This semi-elasticity is based OECD and Treasury calculations (see Price, Dang, and Botev, 2015).
[52] This semi-elasticity is also based OECD and Treasury calculations (see Price, Dang, and Botev, 2015).
[53] I also add cosmetic shocks to smooth the labour and real wage responses.

__Figure 20 — A Large Earthquake__



Net debt to GDP increases by 12 percentage points to 60 percent over the space of 5 years following the earthquake shock, due to higher government spending and lower tax revenues. The total government spending to GDP ratio, which includes increases in transfers payments, government investment and direct EQC payments, is 1.9 percentage points higher than the baseline scenario without an earthquake in 2030. In order to pay down the debt, government needs to raise taxes faster than they do in the baseline scenario. By 2041, tax revenue to GDP is 2.2 percentage points higher than the baseline scenario. The levels of both private and public investment end up 8.5 and 6.6 percent higher relative to the baseline scenario, during the peak of the rebuild phase. This would suggest, based on the spending figures from the Canterbury earthquake, that there is a high degree of “building back better” being captured in the analysis. I compare the GDP projections for a single recession from Section 5.3, against the earthquake scenario in Figure 21.

Figure 21 — GDP Comparison



In the recessions scenario, real GDP falls by 4 percent over 4 quarters to a permanently lower track. In the earthquake scenario, GDP falls by 1.5 percent in the first quarter and then remains elevated for 10 years, due to the rebuilding of infrastructure and the capital stock. After 10 years there is drift down to a GDP track that is 3 percent lower than the baseline scenario, due to higher taxes and a permanently lower total factor productivity track. While the size of the permanent shift in the total factor productivity trend is the same in both scenarios (a 3 percent peak to trough reduction), in the earthquake scenario, the entire fall happens in 1 quarter before total factor productivity growth takes off again. In the recessions scenario, total factor productivity falls by 0.75 percent per quarter for 4 quarters. Because total factor productivity takes longer to reach the trough in the recessions scenario, it ultimately ends up on a lower trajectory than the earthquake scenario, which is reflected in the respective GDP paths.

5.5 Fiscal Consolidation

In this section, I investigate what a fast and a slow fiscal consolidation might look like on top of the baseline ageing population scenario. Under these scenarios government would still commit to funding superannuation and health spending under current settings for the next 40 years, but they would pay down net debt, rather than trying to stabilise it at 48 percent of GDP. Treasury projections suggest that net debt to GDP will more than double to 48 percent over the next 5 years from a pre-COVID level of about 20 percent. All else equal this will have implications for fiscal space and government’s ability to manage future economic and physical shocks. In pre-COVID times the government was aiming for a net debt to GDP target of 20 percent with a $\pm$5 percent band, with Treasury advising that they could raise the debt target to 30 percent over the next decade, so that a prudent level of debt would be a maximum of 50 percent of GDP when factoring in a 20 percent buffer. Post COVID the government has yet to express an explicit net debt to GDP target, although they have stated their intention to stabilise net debt to GDP at current levels and lowering this when conditions permit.[54]

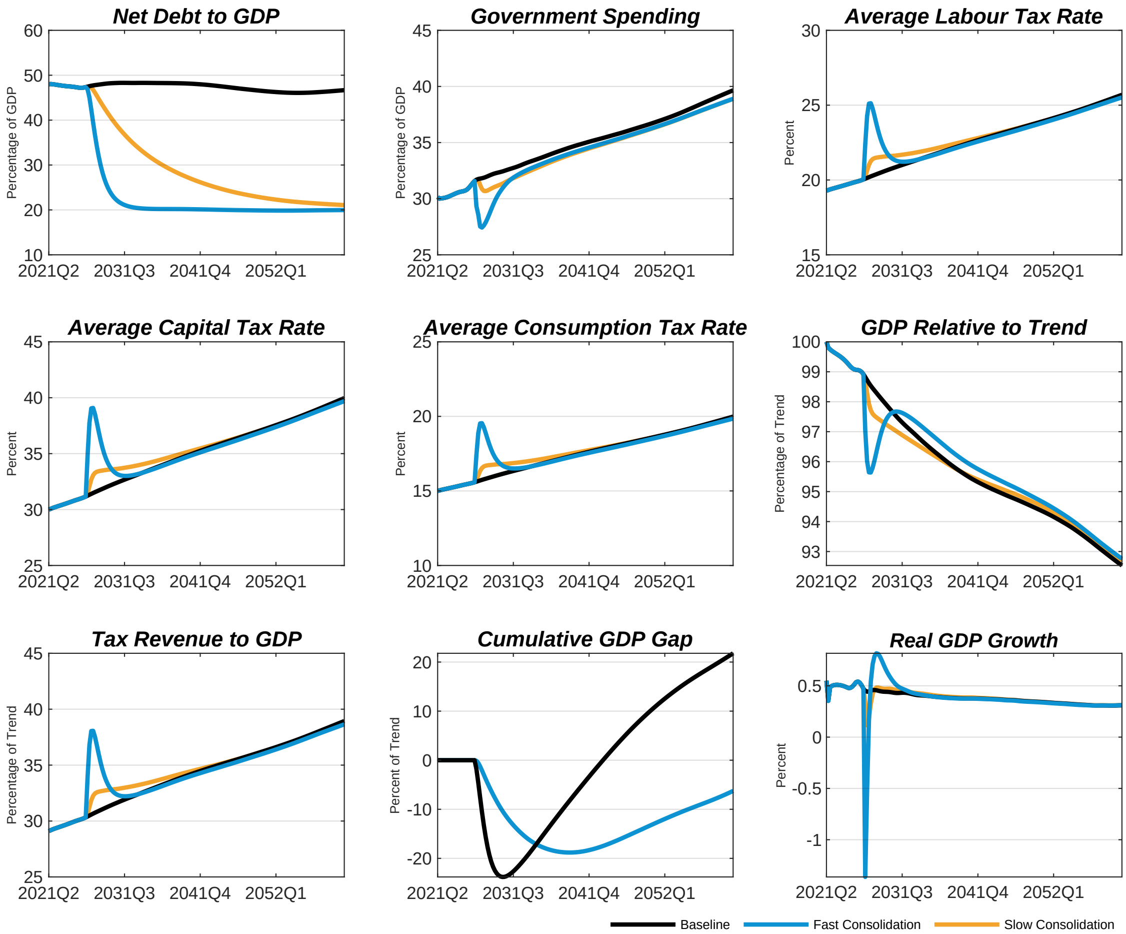
I look at a fast and a slow fiscal consolidation built on top of the baseline projection. Under the fast consolidation, government tries to bring net debt back to 20 percent of GDP over 5 years, while in the slow consolidation they take 35 years to reach the same debt target. Fiscal consolidation requires government to run larger primary surpluses in the short to medium run, as they pay down debt. Once debt is paid down, lower debt servicing costs means government can run smaller primary surpluses. Translating this into fiscal policy, government would need to temporarily run higher taxes, lower government spending or both in order to pay down debt. Once debt is paid down, lower debt servicing costs means government can run lower taxes, higher government spending or both. Higher rates of distortionary taxes and/or lower government spending during the consolidation phase translate to temporarily lower levels of GDP. Once the debt is paid down, lower levels of distortionary taxation and/or higher levels of government expenditure translate to higher levels of GDP. So as not to favour any one of the fiscal instruments over the others, I assume in both consolidation scenarios, government temporarily raises all tax rates and temporarily cuts general government consumption and government investment during the consolidation phase. I do not consider the role that a favourable interest-growth rate differential could play in the result,[55] and it will certainly be the case that different combinations of adjustments in the different fiscal instruments will result in different economic and GDP losses over the consolidation period.[56] And while some of the tax rises and spending cuts may seem implausible, especially in the fast consolidation scenario, they illustrate the magnitudes of the adjustments required to bring down net debt to GDP in the time horizons investigated. Once debt has been lowered, I assume that the lower debt servicing costs are translated into lower tax rates for all tax types, and higher levels of government consumption and investment.

54 The decision for government to consolidate debt is not a simple and straight forward one. Ostry, Ghosh, and Espinoza (2015) show that if a country has sufficient fiscal space, the economic/GDP costs of a fiscal consolidation may not outweigh the economic benefits of a fiscal consolidation, especially if the fiscal consolidation comes at the expense of productive government spending and is financed through distortionary taxation.

55 Eichengreen et al. (2019) investigate a number of consolidation episodes over history and find post World War II negative r minus g played a large role in paying down debt. They attribute this to financial repression, highlighting that it may be more difficult to rely on this channel in the future due to increased financial deregulation.

56 Using empirical methods, Alesina, Favero, and Giavazzi (2019) show that tax based fiscal consolidations are more costly in terms of lost GDP and less likely to succeed. They find that expenditure based fiscal consolidations are less costly in terms of lost GDP and more likely to succeed and in a limited number of cases fiscal consolidations have been stimulatory, due to positive confidence effects encouraging private investment. Fatás and Summers (2018) find that a number of fiscal consolidations undertaken post GFC were self-defeating leaving some countries' net debt to GDP positions in worse shape after the attempted consolidation.

Figure 22 – Fiscal Consolidation



A fast fiscal consolidation would require government to temporarily cut total primary expenditure to GDP by 4 percentage points, keeping total expenditure lower than the baseline for 5 years. At the same time total tax revenue to GDP would need to climb by 7 percentage points relative to the baseline scenario. This in turn would require the tax rates on capital income, labour income and consumption expenditure to spike by 7 percentage points, 5 percentage points and 4 percentage points respectively. The fiscal consolidation would cause a technical recession, with GDP growth falling to -1.5 percent and the cumulative (undiscounted) GDP loss would bottom out at more than 20 percent of the current level of GDP, before turning positive in the 2040s as lower debt servicing costs are translated into lower distortionary tax rates, higher government spending and a higher GDP level. By contrast, the slower fiscal consolidation requires much smaller adjustments in government spending and taxation over a longer period of time. Total government spending as a share of GDP would only need to fall by 1 percentage point and total tax revenue as a share of GDP would only need to increase by 1.5 percentage points. Quarterly real GDP growth would only fall by 0.3 percentage points, which would not be enough to cause a recession in normal times. However the cumulative GDP loss is larger than the fast consolidation in the medium to long run, not even turning positive by the end of the reporting period. This is because shorter, sharper fiscal consolidations are less costly in terms of the total GDP loss, when distortionary taxation, like capital income and labour income taxes are involved. Short sharp adjustments that result in lower taxes in the long run do not give agents the time or the incentive to make large changes to their behaviour. However, smaller changes over longer periods of time give agents more time and more incentive to change their behaviour, which has a proportionately larger impact on GDP.[57]

57 Erceg and Lindé (2013) find a similar result.

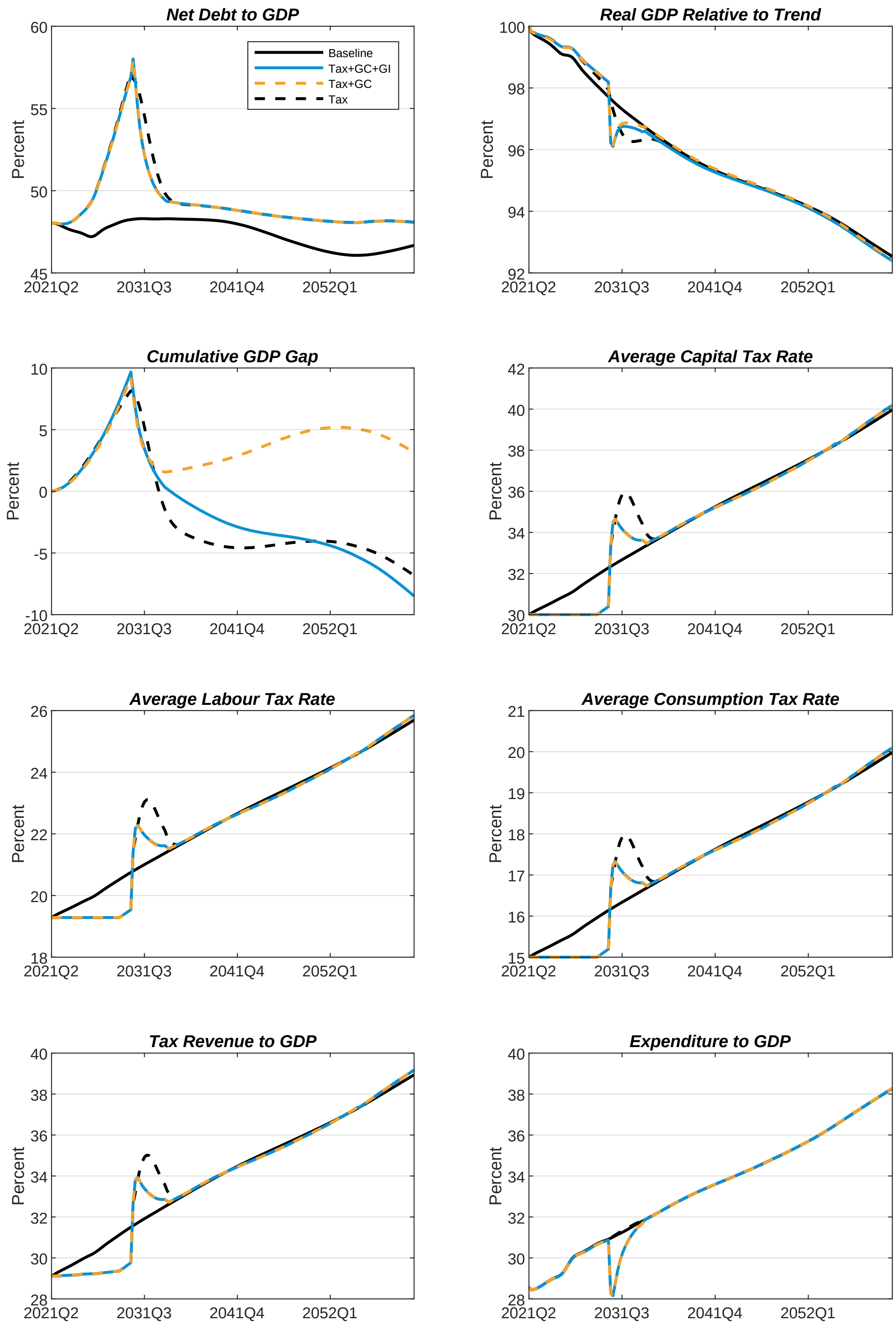
While a short, sharp fiscal consolidation may be less costly in terms of the total GDP loss, this ignores a number of factors, like the intergenerational implications of fiscal consolidations, shorter sharper fiscal consolidations will have a disproportionately larger impact on a very narrow cohort of tax payers and government service users, while a slower fiscal consolidation will have a smaller impact on a much wider cohort of tax payers and government service users. Likewise, deliberately causing a recession may have other costs, not fully captured by the model, that could be larger or more important than the GDP saved from a faster fiscal consolidation.

5.6 A Delayed Response

What happens if government is slow out of the blocks raising taxes to address increasing spending pressures? Or decides in the short run to finance the additional government expenditure through either temporary or permanent increases in debt? I address these questions in this section by looking at two types of delayed response scenarios. In the first, expenses grow for a number of years, while average tax rates are held constant at current levels causing net government debt to increase. Government then decides they want to defend their 48 percent net debt to GDP target by adjusting different combinations of taxation and spending. The second type of scenario starts out like the first, net debt to GDP grows as expenses increase and average tax rates are held constant at current levels for a number of years. However, I investigate what happens if government decides to stabilise net debt at a permanently higher level, and how the timing of the stabilisation affects the results. In both sets of scenarios, increases in government expenditure will be financed by increased borrowing for a time. This will no longer be the case when government decides to either stabilise debt, or consolidate, which will require increased taxation and/or cuts to government expenditure. During this phase of the policy, tax revenue needs to be large enough to cover increased debt servicing costs and pay down debt as well as paying for increased government expenditure due to rising health and superannuation costs.

I run a number of scenarios to investigate the economic costs from a temporary delay in responding to increasing government expenditure, where net debt is allowed to rise, before government introduces a consolidation policy to return net debt to GDP to target. In these scenarios I assume that government keeps average tax rates constant for 7.5 years, which causes net debt to GDP to increase by about 10 percentage points. Government then decides they want to defend the 48 percent net debt to GDP target, by either raising taxes, or through a combination of raising taxes and temporarily cutting government expenditure (combinations of government consumption and government investment). The results for these scenarios are plotted in Figure 23.

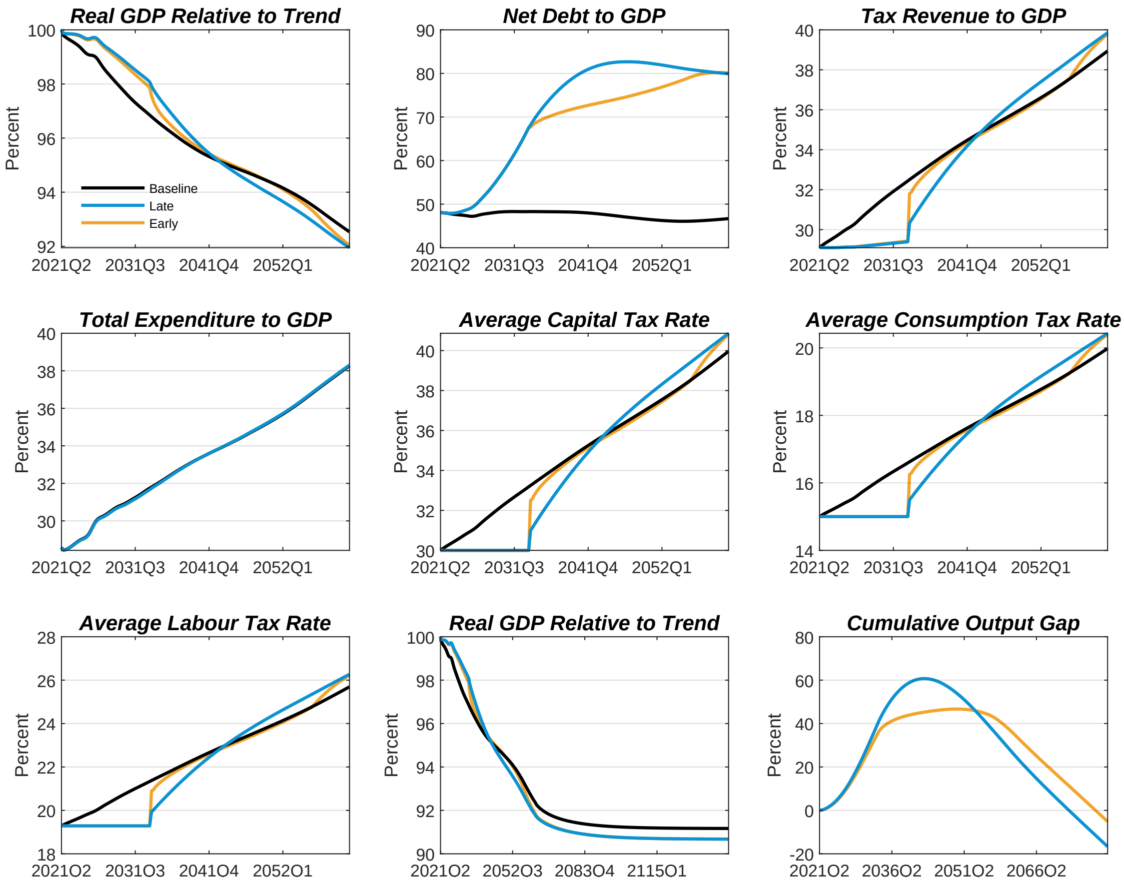
Figure 23 — Expense Stabilization



Real GDP in the alternative scenarios is higher than the baseline scenario for the 7.5 years that government keeps average tax rates constant. This is because government increases their consumption expenditure on health without negatively affecting agents behaviour through increases in distortionary taxation. After 7.5 years government decides they want to stabilise and consolidate net debt, lowering net debt to GDP to 48 percent over a 5 year period. This requires reasonably large adjustments in both average tax rates and government expenditure, when the consolidation is financed through temporary cuts in government expenditure. These adjustments in distortionary taxes and government expenditure cause GDP to dip below the baseline scenario, eventually settling on a path near the baseline once debt has returned to target. The undiscounted cumulative output gap, the percentage difference between the alternative scenario(s) and the baseline, is positive for all alternative scenarios up until the early 2030s. Once the fiscal consolidation starts, GDP and the undiscounted cumulative output gap fall. When the consolidation is financed by increases in tax alone, and a combination of tax increases and cuts to government consumption and government investment, the undiscounted cumulative output gap is negative at the end of the reporting period. When the consolidation is financed through a combination of tax increases and cuts to government consumption, the lost GDP is slightly lower than the GDP gain over the expanding debt phase resulting in a net GDP gain. The difference between these policies reflects the fact that increases in tax rates are more distortionary, costing more in terms of GDP than cuts to government consumption, as changes in government consumption only have an indirect effect on supply. Cuts to government investment result in larger GDP losses when compared with the same size cuts to government consumption because government investment is considered productive, directly affecting aggregate demand and aggregate supply, whereas changes in government consumption only have a direct effect on aggregate demand.

In the second set of scenarios, I compare the baseline against two projections where the tax rates on capital and labour income, and consumption expenditure are held constant for 12.5 years before being allowed to adjust to stabilise net debt at 80 percent of GDP. In the first projection, government responds more aggressively by raising taxes by more earlier to slow the rate of increase in net debt. In the second projection, government's response to the increase in debt is slower allowing debt to increase at a faster rate, resulting in a slight overshoot in net debt to GDP, before it is stabilised at 80 percent. The results from these projections are presented in Figure 24.

Figure 24 — Increased Debt Target: Early vs Late Expense Stabilisation



During the 12.5 year period where tax rates are held constant, net debt to GDP starts to increase at a rapid rate. GDP in the alternative scenarios is higher than the baseline over this period, because there is additional government spending without increases in distortionary taxation. In the early 2030s, government raises taxes drastically to slow the growth in debt. In the scenario labelled “early”, government raises taxes by more early on, to slow the increase in debt, which reaches the new debt target of 80 percent at the end of the reporting period. In the scenario labelled “late”, government initially raises taxes more slowly allowing debt to grow at a faster rate, before being forced to raise taxes by more towards the end of the reporting period, as the higher level of debt requires higher taxes to pay for the higher debt servicing costs. Under both alternative scenarios, real GDP eventually settles at a level that is permanently lower than the baseline projection due to higher distortionary taxation required to fund higher debt servicing costs. The undiscounted cumulative output gap (the difference between the alternative scenarios and the basline) is initially positive for both alternative scenarios during the debt expansion phase of the policies. Under the “late” scenario, the cumulative output gap peaks at a higher level than the output gap in the “early” scenario. This is because net debt reaches a higher level earlier under this scenario. The cumulative output gaps for both scenarios decline, eventually turning negative as the costs of higher taxation weigh on the economy. Both alternative scenarios illustrate that there are short-term gains from raising debt to fund government consumption, but in the long run higher debt servicing costs, which leads to higher taxation, results in lower GDP levels, relative to trend.

6. Climate Change: Storms and Droughts Scenarios

In this section I look at the effect of extreme weather events, namely droughts and storms, on the economy and the government's fiscal position. I produce a couple of scenarios that look at the implications of sequences of weather events arriving at random intervals. I factor in climate change by either increasing the frequency or the intensity of droughts and storms over the reporting period. The reporting period extends to 2061, although the impacts of climate change will last well beyond this period and could continue to worsen. The weather scenarios are kept separate from the ageing population scenarios for practical purposes and ease of interpretation.[58]

Modelling the impact of climate change on the macroeconomy and the fiscal position brings with it a number of challenges, including a high degree of uncertainty. I limit analysis to a narrowly defined area of climate change under strict assumptions. I do not consider the impact of climate change mitigation and associated policies on the scenarios. Nor do I explicitly capture the effects of adaptation in these scenarios. Given the time horizon I am dealing with it is likely there will be some adaption which could reduce the impact and damage from particular weather events. Furthermore, I assume that climate change does not change the current trend or balanced growth path. It would be reasonable to assume that warmer temperatures lower trend total factor productivity.[59] I do not model the impact of climate change or extreme weather events in the rest of the world. An increase in the frequency and intensity of extreme weather events globally could have a non-trivial impact on trade and migration flows to New Zealand. These omissions mean that the scenarios may understate the overall impact of climate change.

6.1 Droughts

New Zealand is a large producer and exporter of primary produce. New Zealand's favourable climate and geography mean it has a comparative advantage producing products like wool, meat, dairy products, wood and timber products, and seafood. Extended periods of adverse weather, including droughts, have negative economic consequences for New Zealand. It's widely held that drought was a contributing factor in the 1998 recession (see OECD, 2000; IMF, 2003; Reddell and Sleeman, 2008, for example). Climate change brings with it the likelihood that the frequency and severity of droughts will increase with time. NIWA (2011) predicts the frequency and duration of droughts could increase by about 10 percent by 2040, which for many areas would see a doubling of the duration and frequency of their time spent in drought. This will have consequences for the New Zealand economy and the country's fiscal position. A drought could potentially lower the tax base as production and incomes fall. Government may need to spend more to stimulate the economy, while paying out more in unemployment benefits if there are job losses. I investigate the potential economic implications of drought over the 40 year reporting horizon through repeated stochastic simulation. In particular I look at the implications of the expected increase in the frequency of droughts over the next 40 years on net government debt to GDP and GDP relative to trend.

I construct simulations by treating droughts as discrete events of fixed sizes that arrive according to a stochastic process. It is natural to think of droughts as discrete events, at any point in time a region or a country either experiences drought or it does not. Treating droughts this way also enables the discretionary fiscal response to be calibrated accordingly.[60] I consider droughts of two particular sizes, (current) 1 in 10 year events and (current) 1 in 20 year events, with half

[58] This means I leave out all the exogenous demographic adjustments that are predicted to occur over the reporting period. I also change the government's spending rules so that they are in levels, rather than ratios, which makes their interpretation and adjustment easier in the absence of long-run trends.

[59] This is the usual assumption used when considering damage functions.

[60] This means adding sequences of government spending or transfers shocks

of all 1 in 10 year events being at least as large as a 1 in 20 year event. I use the work of Gallic and Vermandel (2019), Buckle et al. (2007) and Kamber, McDonald, and Price (2013) to calibrate these scenarios. Kamber, McDonald, and Price (2013) find the 2012 drought, a 1 in 20 year event, lowered GDP at the trough by about 0.7 percent. Buckle et al. (2007) find the 1998 drought, which was the second worst drought to hit the North Island in the 1972-2013 period, lowered GDP by about 1 percent relative to trend.[61] For the stochastic simulations I assume that a 1 in 20 year event lowers GDP by about 1 percent and a 1 in 10 year event lowers GDP by about 0.5 percent. I lower GDP in both events by adding a sequence of negative productivity growth shocks for 3 quarters. Gallic and Vermandel (2019) construct a two sector structural model of the New Zealand economy and model droughts as negative productivity shocks in the primary production sector. I assume that government adds extra stimulus, spending 2.5 percent percent cumulatively (in annualised terms) over 3 quarters in the 1 in 20 year event and 1.5 percent in the 1 in 10 year event. This is done by adding general government spending shocks in the periods after the drought hits. I choose these numbers so they are roughly in line with the estimated government responses in Kamber, McDonald, and Price (2013). I assume that government responds reasonably aggressively to deviations of net government debt from its target by raising labour income tax rates, so that net debt to GDP has an average half life of about 3 years. The responses for a single 1 in 10 year and 1 in 20 year drought can be found in Figure 31 in Appendix J.

While each drought event is calibrated to match the current impact on the economy, going forward these numbers could change depending on New Zealand's reliance on sectors and industries that are more heavily affected by drought. Droughts could become more intense and last for longer as temperatures rise. This could be modelled by adding larger negative productivity shocks over a longer period or by increasing the persistence of the productivity process. At the same time, adaptation, changing crops, animals and feed types could increase the primary sector's resilience to the effects of drought. The movement away from more drought affected industries into less drought affected industries could also improve the economy's drought resilience. This increased resilience can be modelled by using smaller negative productivity shocks to represent droughts.

I assume that the frequency of droughts doubles by 2040 from current levels, and triples by 2060, which is in line with NIWA projections (NIWA, 2011). This means that what is currently a 1 in 20 year event will become a 1 in 10 year event by 2040, and what is currently a 1 in 10 year event will be a 1 in 5 year event by 2040.[62] Because these are treated as discrete events, their arrival can be modelled using a Poisson process under the assumption that the probability of events occurring is independent and that they do not occur simultaneously. The frequency of droughts is expected to increase with time, so I abandon the usual homogeneity assumption and instead draw events from a non-homogeneous Poisson process.[63] As half the 1 in 10 year events are also 1 in 20 year events, I can use the non-homogenous Poisson process for current 1 in 10 year events that increase in frequency to also model the 1 in 20 year events that increase in frequency. When a drought event is encountered I "flip a coin" to determine whether the event is the size of a current 1 in 10 year event, or the size of a current 1 in 20 year event.[64] The full

[61] This is second worst drought based on PED (potential evapotranspiration deficit) measures (see MPI, 2013).

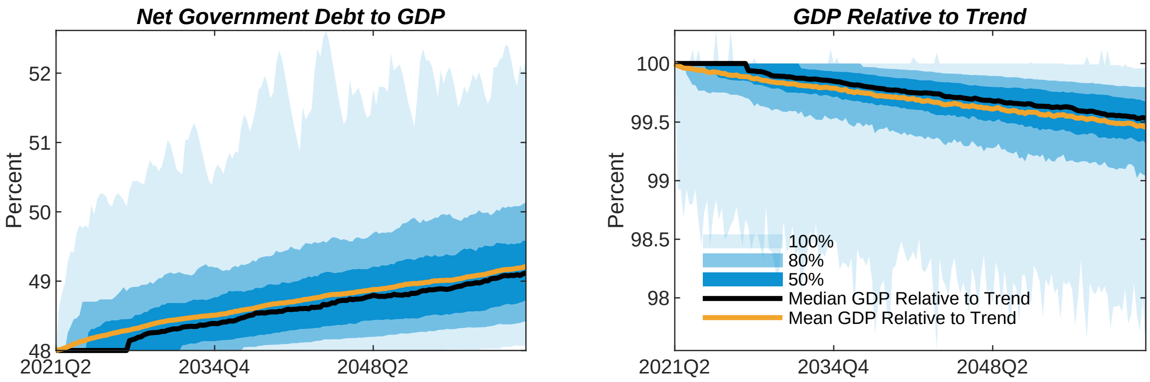
[62] I do not take into account how climate change may also increase the frequency and duration of droughts in other countries. Factoring in an increased frequency of droughts in competitors' countries could mitigate some of the effects of drought by pushing up global agricultural prices.

[63] Alternatively I could have considered modelling the drought process as a compound Poisson Gamma distribution or a compound Poisson Log-normal distribution, where the Poisson arrival rate is a function of the intensity of the drought. Using a compound Poisson distribution would allow both discrete events and their magnitudes to be drawn from the same stochastic process. However, this would also require calibrating the process adding to the complexity of the exercise, so for the moment I keep things relatively simple by modelling droughts as being of a fixed size and duration and just consider changes in drought frequency.

[64] "Coin flipping" draws are made by drawing from a discrete uniform distribution, with draws below 0.5 designated 1 in 10 year events, and draws above 0.5 designated 1 in 20 year events.

distribution for the model variables is calculated through repeated stochastic simulation. I run 1000 stochastic simulations and construct fan charts for net debt to GDP and GDP relative to trend, which are presented in Figure 25 below. The bands represent the 50th, the 80th and 100th percentiles of the marginal distributions.

Figure 25 – Droughts



According to the modelling exercise, an increase in the frequency of droughts over the next 40 years lowers GDP by 0.5 percent and raises net debt to GDP by more than 1.2 percentage points on average. Particularly bad sequences of shocks could lower GDP by more than 2 percent and increase net debt to GDP by more than 14 percentage points, as back to back droughts, occuring in a short space of time compound the situation. The overall implications for net debt to GDP are relatively small, although this depends crucially on the assumptions made about the fiscal policy response to droughts. The recessions in Section 5.3 raised net debt to GDP by between 11 to 13 percentage points from pre-recession levels and by 16 percentage points when compared with the no recession counterfactual for a 4 percent (peak to trough) decline in GDP. Doing some rough back of the envelop calculations and multiplying the average increase in net debt to GDP in the stochastic droughts scenario by 8, to make the decline in GDP comparable with the recessions scenario, gives a scaled net debt to GDP response of 9.6 percentage points, which is not too different from the debt response in the recessions scenario.

6.2 Storms

Extreme weather events like storms, cyclones, floods and heavy rainfall episodes are predicted to increase in strength as global temperatures rise (MfE, 2018).[65] Warmer air holds more water vapour resulting in more precipitation, while warmer sea temperatures could mean ex-tropical cyclones and storms lose less energy or intensity as they move south toward New Zealand. These weather events can be particularly destructive to the capital stock. Roads, bridges, railway tracks, along with private property can be washed out, destroyed or written off with floods, rising rivers and storm surges. Heavy rain also increases the chance of landslips. This has flow on consequences to insurance premiums and access to insurance. I produce stochastic simulations to illustrate the possible effects of storm or flood events on the fiscal position of the economy over the next 40 years, I report the results as fan charts for net debt to GDP and GDP relative to trend.

65 Globally, storms are predicted to increase in intensity as temperatures increase with climate change. What this means for the frequency and intensity of ex-tropical cyclones that make landfall in New Zealand is a little more uncertain. However, extreme rainfall and wind events in New Zealand are predicted to increase in intensity with climate change (MfE, 2018).

The storms scenario is constructed under similar assumptions to the droughts scenario with discrete weather events arriving according to a stochastic process. The storm event is characterised by a capital destruction "shock" on both the publicly and the privately owned capital stocks, as was used in the earthquake scenario in Section 5.4.[66] Fornero and Kirchner (2018) have used the same approach to model the impact of earthquakes in Chile. Wright and Borda (2016) have used a similar approach to model natural disasters (hurricanes, storms, earthquakes) more generally in the Carribean and Central America.[67] The shock occurs in the period when the weather event impacts the economy and there are parameters that can be set to determine how much of the publicly and privately owned capital stock is destroyed. I also assume that after the shock has hit the economy, government increases consumption and investment spending to stimulate the economy and rebuild the destroyed capital stock.

Determining the cost of natural disasters and the amount of the nation's capital stock destroyed in the disaster provides some challenges. Exact costs of historical natural disasters, especially the quantity of the capital stock destroyed, are hard to come by. Typically the costs of natural disasters are constructed using private insurance and government spending costs (see ECRC, 2018, for example). Differences in methodology mean that estimates can vary substantially. Furthermore this may understate costs due to under insurance and non-insurance. Historically the cost of floods, as measured by insurance costs, have been low in New Zealand relative to the size of GDP and the capital stock. Frame et al. (2018) put the cost of floods based on private insurance data at $406.5 million, of which only $120 million can be attributed to climate change, for the decade between 2007 and 2017. While these costs do not include government spending, they pale in comparison with the Canterbury earthquakes which cost more than $40 billion (Wood, Noy, and Parker, 2016), including more than $15 billion in government spending (The New Zealand Treasury, 2014).[68] Statistics New Zealand estimates that $14.8 billion of the capital stock was destroyed or written off in the Canterbury earthquakes, or approximately 2 percent of the nation's capital stock at the time.

While the historical cost of storms and cyclones has been low, increases in the size and frequency of storms, combined with aggravating factors like higher sea levels and increased development or population, could make the economic impact of storms worse in the future. The evidence on how climate change could affect the frequency of ex-tropical cyclones is unclear, but it is believed that climate change will intensify extreme rainfall events (MfE, 2018). Determining the expected cost of future storm events is difficult. While storm size is an important factor, where the storm makes landfall is just as important. If a storm of a given intensity makes landfall in a sparsely populated area of the country, the damage to infrastructure and the capital stock is likely to be lower than if it makes landfall in a highly populated area. Due to the low lying nature or proximity to rivers, some areas are more vulnerable to the impact of storms and intense precipitation than other areas. An increase in the population could make storms more costly, as storms over more populated or developed areas tend to cause more damage to property and infrastructure. However, if the effects of climate change are factored into planning and design decisions, some of the impact of the weather events can be offset or reduced, so that they are less costly.[69] Flooding is the biggest cost for storm and cyclone related events. Currently $12.5 billion in assets in New Zealand are at risk of storm surge, while a 0.3 meter increase in the sea level sees this increase to $18.5 billion, and a 0.6 meter increase means that $26.2 billion in assets are at risk (NIWA, 2019a). Currently there are $135 billion in assets in New Zealand at risk from either river and/or storm surge flooding (NIWA, 2019b), which is approximately 16

66 For the storms scenario, I do not assume that trend productivity is affected by the weather event.

67 A similar approach has been used by Isoré (2018) to model natural disaster risk in selected Latin American countries and by Keen and Pakko (2007) to model the Fed's response to Hurricane Katrina.

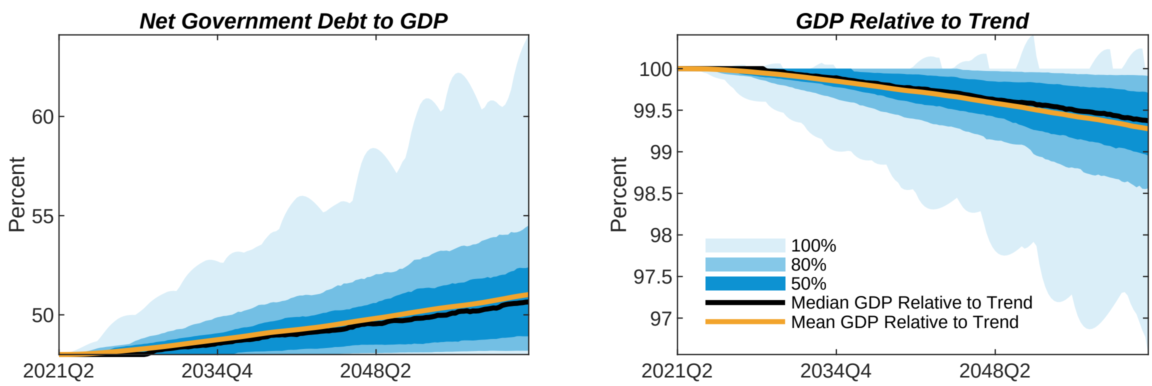
68 In the earthquake scenario from Section 5.4, I assume that government spent $7 billion on investment and $11.8 billion on EQC payouts.

69 This could include building infrastructure that is more resilient to extreme weather events, or moving the population to areas of the country that are less affected by extreme weather events.

percent of the nation's capital stock. Australia, due to its size and geographic proximity, is hit more regularly by intense storms than New Zealand. The cost of cyclone Oswald was estimated to be $2.3 billion AUD,[70] while cyclone Debbie is estimated to have cost $3.5 billion AUD.[71] If Australia is used as the reference point for the future, and similar sized storms are expected to occur in New Zealand over areas with a similar population size and density, then it is quite plausible that New Zealand could be hit by storms that destroy 0.5 percent of the capital stock. Because this is a stress testing exercise of sorts, I assume in the stochasic simulations that 1 in 10 year storms and floods destroy 0.2 percent of the capital stock in 2021, with the frequency of events fixed, but the intensity of events increasing over the 40 year reporting period, so that 1 in 10 year storms destroy 2 percent of the capital stock by 2061. The government's spending response is adjusted proportionately to the size of the storm, using the peak fiscal response to the Canterbury earthquakes as a reference point. Labour taxes are assumed to respond reasonably aggressively to deviations of net debt from target implying an average half life of just over 6 years. Figure 32 in Appendix J depicts storm events that destroy 0.2 percent and 2 percent of the nations capital stock in the first period.

Applying a similar approach used in the droughts scenario, I assume that storms arrive at random intervals over the projection period according to a Poisson process. Unlike the droughts scenario, the frequency of arrival is assumed to remain constant over the reporting horizon, which means the events can be drawn from a homogeneous Poisson process. Kossin, Camargo, and Sitkowski (2010) have modelled the arrival rate of hurricanes in the North Atlantic as a Poisson process. I set the arrival rate for the Poisson process to 1/40 per quarter, so that 4 storms, on average, are expected over the 40 year (160 quarter) reporting horizon. This is an arbitrary choice, given there is not a lot of information about the expected frequency of storms over this period. Distributions for the model variables are calculated through repeated stochastic simulation. I generate 1000 draws from the Poisson process, when I reach a time period where a storm event occurs, I shock the model with a capital destruction shock and add positive government consumption and investment shocks to capture the government's response. The results are reported as fan charts in Figure 26 below.

Figure 26 – Storms



If 1 in 10 year storms increase in size, so that they are expected to destroy 2 percent of the nation's capital stock by 2061, net debt to GDP will be about 3 percentage points higher and GDP will be more than 0.7 percent lower than trend, on average according to the model. Particularly bad sequences of shocks could raise net debt to GDP by more than 15 percentage points and lower GDP by more than 3 percent. Scaling the net debt to GDP response by the size of the reduction in GDP so that it can be compared against the recessions scenario means multiplying

70 QEAS (2017).
71 IAN (2019).

the 3 percent net debt to GDP response by 4/0.7 to get 17 percentage points. This is a bit larger than the response in the recessions scenario, where net debt to GDP increased by between 11 percent and 13 percentage points relative to pre-recession levels. Scaling the net debt to GDP response to match the earthquake scenaro would mean multiplying the 3 percent net debt to GDP response by 1.5/0.7 to get 6.4 percentage points. This is lower than the 12 percentage point response in the earthquake scenario.

7. Conclusion

I develop a reasonably simple stochastic neoclassical growth model in this paper, which I use to produce shocks and scenarios analysis to support the combined long-term fiscal statement and insights briefing. The long-term fiscal statement, which the Treasury is normally required to produce once every four years, outlines how the Crowns fiscal position could evolve over the next 40 years, while the long-term insights briefing includes analysis and policy options that governments could use to respond to future fiscal challenges. I produce projections and scenarios to illustrate some of the fiscal and economic implications of an ageing population and increased health spending. I also use the model to look at some narrowly defined aspects of climate change, namely an increased frequency and severity of extreme weather events. In developing and using the NCGM I address some of the issues raised about the analysis in previous long-term fiscal statements and the use of spreadsheet accounting models in general, in particular the lack of feedback or behavioural responses in these models and the absence of shocks and uncertainty analysis in the results. The simplifying assumptions required to make the NCGM tractable mean the fiscal block is more aggregated and the model more stylised than the LTFM in general. As a consequence the analysis from the NCGM should be viewed as complementary to the LTFM, with some of the NCGM's key inputs coming from the LTFM.

Under the baseline ageing population scenario, government commits to funding all projected expenditure increases by raising taxes each period to balance the budget and defend the debt target. While government is able to meet all their expenses, this requires a substantial increase in tax rates by the end of the reporting period, with tax revenues needing to increase from 29.1 percent of GDP in 2021 to 38.9 percent of GDP in 2061 to balance the budget. And although increased health spending provides a partial offset to increases in distortionary taxation, GDP still ends up 3.7 percent lower than a world where expenses are fixed at their current GDP share. In addition to the baseline projection, I investigate a number of alternative scenarios built on top of the ageing population scenario. These include:

- *Recessions*: I investigate a series of uniformly spaced recessions constructed to match stylised recession facts for New Zealand. Recessions add between 11 and 13 percentage points to net debt to GDP. Government can consolidate net debt to GDP and continue to fund expenses, but this requires a reasonably aggressive tax policy response.

- *An Earthquake*: I construct a scenario to resemble an earthquake that is 50 percent more destructive than the Canterbury earthquakes to represent a large earthquake in Wellington. Net debt increases by 12 percentage points and is assumed to take more than a decade to get back to target. GDP eventually shifts to a track that is 3 percent lower than the baseline projection.

- *Fiscal Consolidation*: I investigate two fiscal consolidation scenarios, a fast consolidation and a slow consolidation. A fast consolidation causes more disruption in the short run, having a larger impact on the cohort of tax payers/government service users alive at that time, but has a more positive impact in the medium to long run due to lower debt servicing costs. By contrast a slower consolidation has a smaller average impact over a longer period of time, affecting a wider cohort of tax payers and government service users. Because debt is slower to come down, higher debt servicing costs lead to slightly lower GDP, which adds up over a long period of time.

- *A Delayed Response*: I investigate a number of delayed policy response scenarios, where expenditure continues to grow, but government is slow in raising taxes to stabilise or consolidate net debt. The scenarios show that in the short run there may be some economic benefit allowing government debt to increase to pay for the increase in expenditure. However once government decides they want to stabilise or consolidate debt, higher distortionary taxes due to higher debt servicing costs or the larger primary surpluses that need to be run to pay down debt, end up dominating any short-term benefits from running up debt.

Focusing on a very narrow range of climate change issues, I investigate the fiscal and economic implications of increases in the frequency and intensity of extreme weather events over the 40 year reporting period. Modelling weather events as discrete events that arrive according to a Poisson process, increases in the frequency of droughts are expected to raise net debt to GDP by 1.2 percentage points and lower the level of GDP by 0.5 percent relative to trend, on average. Increases in the severity of storms lead to a 3 percentage point increase in net debt to GDP and a 0.7 percent fall in GDP relative to trend under the assumptions considered.

I make a number of contributions, both technical and practical, in this paper. This is the first time Treasury has used a general equilibrium model to produce shocks and scenarios analysis for the long-term fiscal statement. I demonstrate how the fiscal impacts of an ageing population and some aspects of climate change can be modelled in such a framework. I develop Matlab codes for solving dynamic macroeconomic models on a deterministic transition path and using the extended path algorithm. This includes the ability to incorporate conditional forecasting with exactly identified shocks. I develop a novel method for investigating the economic implications of extreme weather events that is applicable to a broad class of dynamic macroeconomic models. While I did not consider an overlapping generations framework this time, future research should look at producing long-term projections using overlapping generations models, with the possibility of using them in future long-term fiscal statements.

References

Aiyagari, S. Rao, Lawrence J. Christiano, and Martin Eichenbaum (1992). "The Output, Employment, and Interest Rate Effects of Government Consumption". In: *Journal of Monetary Economics* 30.1, pp. 73–86. URL: https://ideas.repec.org/a/eee/moneco/v30y1992i1p73-86.html.

Alesina, Alberto, Carlo Favero, and Francesco Giavazzi (2019). *Austerity: When It Works and When It Doesn't*. Princeton University Press. URL: https://press.princeton.edu/books/hardcover/9780691172217/austerity.

Alinaghi, Nazila and W. Robert Reed (2021). "Taxes and Economic Growth in OECD Countries: A Meta-analysis". In: *Public Finance Review* 49.1, pp. 3–40. URL: https://ideas.repec.org/a/sae/pubfin/v49y2021i1p3-40.html.

Andrle, Michal, Jan Bruha, and Serhat Solmaz (2016). *On the Sources of Business Cycles: Implications for DSGE Models*. Working Papers 2016/03. Czech National Bank. URL: https://ideas.repec.org/p/cnb/wpaper/2016-03.html.

Attanasio, Orazio, Sagiri Kitao, and Giovanni Violante (2010). "Financing Medicare: A General Equilibrium Analysis". In: *Demography and the Economy*. National Bureau of Economic Research, Inc, pp. 333–366. URL: https://EconPapers.repec.org/RePEc:nbr:nberch:8420.

Auerbach, Alan J. (1985). "The Theory of Excess Burden and Optimal Taxation". In: *Handbook of Public Economics*. Ed. by A. J. Auerbach and M. Feldstein. Vol. 1. Handbook of Public Economics. Elsevier. Chap. 2, pp. 61–127. URL: https://ideas.repec.org/h/eee/pubchp/1-02.html.

Auerbach, Alan J. and Laurence J. Kotlikoff (1988). *Dynamic Fiscal Policy*. Cambridge University Press. URL: https://books.google.co.nz/books?id=FghdvwEACAAJ.

Backus, David, Patrick J. Kehoe, and Finn E. Kydland (1993). *International Business Cycles: Theory and Evidence*. Working Paper 4493. National Bureau of Economic Research. URL: http://www.nber.org/papers/w4493.

Baksa, Daniel and Zsuzsa Munkacsi (2016). *A Detailed Description of OGRE, the OLG Model*. Bank of Lithuania Working Paper Series 31. Bank of Lithuania. URL: https://ideas.repec.org/p/lie/wpaper/31.html.

Baksa, Daniel, Zsuzsa Munkacsi, and Carolin Nerlich (2020). *A Framework for Assessing the Costs of Pension Reform Reversals*. Working Paper Series 2396. European Central Bank. URL: https://ideas.repec.org/p/ecb/ecbwps/20202396.html.

Ball, Christopher, John Creedy, and Grant Scobie (2015). *Long-run Fiscal Projections Under Uncertainty: The Case of New Zealand*. Tech. rep.

Barksby, Laura (2021). *NZ: Insurance Resilience 10 Years After the Christchurch Earthquake*. URL: https://www.preventionweb.net/news/view/76265.

Barro, Robert J. (1980). *Output Effects of Government Purchases*. NBER Working Papers 0432. National Bureau of Economic Research, Inc. URL: https://ideas.repec.org/p/nbr/nberwo/0432.html.

— (2006). "Rare Disasters and Asset Markets in the Twentieth Century". In: *The Quarterly Journal of Economics* 121.3, pp. 823–866. URL: https://ideas.repec.org/a/oup/qjecon/v121y2006i3p823-866..html.

Baxter, Marianne and Robert G. King (1993). "Fiscal Policy in General Equilibrium". In: *American Economic Review* 83.3, pp. 315–334. URL: https://ideas.repec.org/a/aea/aecrev/v83y1993i3p315-34.html.

Bell, Matthew (2021). *Demographic, Economic and Fiscal Assumptions and Logic in the 2021 Long-Term Fiscal Model*. Background Paper for 2021 Statement on the Long-Term Fiscal Position. The New Zealand Treasury.

Bell, Matthew and Melissa Piscetek (2016). *Demographic, Economic and Fiscal Assumptions and Modelling Methods in the 2016 Long-Term Fiscal Model*. Background Paper for 2016 Statement on the Long-Term Fiscal Position. The New Zealand Treasury. URL: https://treasury.govt.nz/sites/default/files/2016-11/ltfs-16-bg-defamm.pdf.

Benati, Luca and Thomas Lubik (2021). *Searching for Hysteresis*. Diskussionsschriften dp2107. Universitaet Bern, Departement Volkswirtschaft. URL: https://ideas.repec.org/p/ube/dpvwib/dp2107.html.

Bi, Huixin (2012). "Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy". In: *European Economic Review* 56.3, pp. 389–410. URL: https://EconPapers.repec.org/RePEc:eee:eecrev:v:56:y:2012:i:3:p:389-410.

Bielecki, Marcin, Michał Brzoza-Brzezina, and Marcin Kolasa (2018). *Demographics, Monetary Policy, and the Zero Lower Bound*. Tech. rep.

Binning, Andrew (2021). *An Efficient Application of the Extended Path Algorithm in Matlab With Some Examples*. Note.

Blanchard, Olivier (2019). "Public Debt and Low Interest Rates". In: *American Economic Review* 109.4, pp. 1197–1229. URL: https://ideas.repec.org/a/aea/aecrev/v109y2019i4p1197-1229.html.

— (2021). *Fiscal & Monetary Policy after COVID*. https://bcf.princeton.edu/wp-content/uploads/2021/03/Combined-Slides-2.pdf. Accessed: 202109-11.

Borger, Christine, Thomas F. Rutherford, and Gregory Y. Won (2008). "Projecting Long Term Medical Spending Growth". In: *Journal of Health Economics* 27.1, pp. 69–88. URL: https://ideas.repec.org/a/eee/jhecon/v27y2008i1p69-88.html.

Boucekkine, Raouf (1995). "An Alternative Methodology for Solving Nonlinear Forward-Looking Models". In: *Journal of Economic Dynamics and Control* 19.4, pp. 711–734. URL: https://ideas.repec.org/a/eee/dyncon/v19y1995i4p711-734.html.

Bry, Gerhard and Charlotte Boschan (1971). *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*. NBER. URL: http://www.nber.org/books/bry_71-1.

Buckle, Robert A. (2018). *A Quarter of a Century of Fiscal Responsibility: The Origins and Evolution of Fiscal Policy Governance and Institutional Arrangements in New Zealand, 1994 to 2018*. Working Paper Series 7693. Victoria University of Wellington, Chair in Public Finance. URL: https://ideas.repec.org/p/vuw/vuwcpf/7693.html.

Buckle, Robert A., Kunhong Kim, Heather Kirkham, Nathan McLellan, and Jarad Sharma (2007). "A Structural VAR Business Cycle Model for a Volatile Small Open Economy". In: *Economic Modelling* 24.6, pp. 990–1017. URL: https://ideas.repec.org/a/eee/ecmode/v24y2007i6p990-1017.html.

CBO (2012). *How the Supply of Labor Responds to Changes in Fiscal Policy*. Congressional Budget Office Report. URL: https://www.cbo.gov/sites/default/files/cbofiles/attachments/10-25-2012-Labor_Supply_and_Fiscal_Policy.pdf.

— (2021). *The 2021 Long-Term Budget Outlook*. Congressional Budget Office Report. URL: https://www.cbo.gov/system/files/2021-03/56977-LTBO-2021.pdf.

Cerra, Valerie, Antonio Fatás, and Sweta Saxena (2020). *Hysteresis and Business Cycles*. CEPR Discussion Papers 14531. C.E.P.R. Discussion Papers. URL: https://ideas.repec.org/p/cpr/ceprdp/14531.html.

Cerra, Valerie and Sweta Chaman Saxena (2003). *Did Output Recover From the Asian Crisis?* IMF Working Papers 2003/048. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2003-048.html.

— (2008). "Growth Dynamics: The Myth of Economic Recovery". In: *American Economic Review* 98.1, pp. 439–457. URL: https://ideas.repec.org/a/aea/aecrev/v98y2008i1p439-57.html.

— (2017). *Booms, Crises, and Recoveries: A New Paradigm of the Business Cycle and its Policy Implications*. IMF Working Papers 2017/250. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2017-250.html.

Chari, V. V, Patrick J. Kehoe, and Ellen R. McGrattan (2002). "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" In: *Review of Economic Studies* 69.3, pp. 533–563. URL: https://ideas.repec.org/a/oup/restud/v69y2002i3p533-563.html.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011). "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive

Margins". In: *American Economic Review* 101.3, pp. 471–475. URL: https://ideas.repec.org/a/aea/aecrev/v101y2011i3p471-75.html.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy". In: *Journal of Political Economy* 113.1, pp. 1–45. URL: https://ideas.repec.org/a/ucp/jpolec/v113y2005i1p1-45.html.

Cogan, John F., Tobias Cwik, John B. Taylor, and Volker Wieland (2010). "New Keynesian Versus Old Keynesian Government Spending Multipliers". In: *Journal of Economic Dynamics and Control* 34.3, pp. 281–295. URL: https://ideas.repec.org/a/eee/dyncon/v34y2010i3p281-295.html.

Coleman, Andrew (2019). *Taxing Capital Income in New Zealand: An International Perspective*. Discussion Paper 1902. University of Otago. URL: https://ourarchive.otago.ac.nz/bitstream/handle/10523/8789/DP_1902.pdf?sequence=1&isAllowed=y.

Conway, Paul, Lisa Meehan, and Dean Parham (2015). *Who Benefits from Productivity Growth? - The Labour Income Share in New Zealand*. Working Paper 2015/1. The New Zealand Productivity Commission. URL: https://www.productivity.govt.nz/assets/Documents/the-labour-income-share-in-new-zealand/e6a1d17058/Report-The-labour-income-share-in-New-Zealand.pdf.

Cooley, Thomas (1997). "Calibrated Models". In: *Oxford Review of Economic Policy* 13.3, pp. 55–69. URL: https://EconPapers.repec.org/RePEc:oup:oxford:v:13:y:1997:i:3:p:55-69.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc (2008). "High Exchange-Rate Volatility and Low Pass-through". In: *Journal of Monetary Economics* 55.6, pp. 1113–1128. URL: https://ideas.repec.org/a/eee/moneco/v55y2008i6p1113-1128.html.

Cousins, Jim, Natalia Deligne, and Mostafa Nayyerloo (2014). *Estimated Damage and Casualties from Earthquakes Affecting Auckland*. GNS Science Consultancy Report 2013/324. Institute of Geological and Nuclear Sciences Limited. URL: https://www.researchgate.net/publication/266477223_Estimated_damage_and_casualties_from_earthquakes_affecting_Auckland.

Cousins, Jim, Andrew King, and Marlene Kanga (2012). *Accumulated Losses from Sequences of Earthquakes*. 15th World Conference on Earthquake Engineering, 24-28 September, Lisbon, Portugal. URL: https://www.iitk.ac.in/nicee/wcee/article/WCEE2012_3812.pdf.

Creedy, John (2003). *The Excess Burden of Taxation and Why it (Approximately) Quadruples When the Tax Rate Doubles*. Treasury Working Paper Series 03/29. New Zealand Treasury. URL: https://ideas.repec.org/p/nzt/nztwps/03-29.html.

Creedy, John and Thai-Yoong Mok (2017). *Labour Supply Elasticities in New Zealand*. Working Paper Series 6570. Victoria University of Wellington, Chair in Public Finance. URL: https://EconPapers.repec.org/RePEc:vuw:vuwcpf:6570.

Creedy, John and Grant Scobie (2017). "Debt Projections and Fiscal Sustainability with Feedback Effects". In: *New Zealand Economic Papers* 51.3, pp. 237–261. URL: https://ideas.repec.org/a/taf/nzecpp/v51y2017i3p237-261.html.

Creedy, John, Norman Gemmell, Nicolas Hérault, and Penny Mok (2018). *Microsimulation Analysis of Optimal Income Tax Reforms: An Application to New Zealand*. Melbourne Institute Working Paper Series wp2018n07. Melbourne Institute of Applied Economic and Social Research, The University of Melbourne. URL: https://ideas.repec.org/p/iae/iaewps/wp2018n07.html.

Davig, Troy, Eric M. Leeper, and Todd B. Walker (2010). ""Unfunded Liabilities" and Uncertain Fiscal Financing". In: *Journal of Monetary Economics* 57.5, pp. 600–619. URL: https://ideas.repec.org/a/eee/moneco/v57y2010i5p600-619.html.

De Paoli, Bianca (2009). "Monetary Policy under Alternative Asset Market Structures: The Case of a Small Open Economy". In: *Journal of Money, Credit and Banking* 41.7, pp. 1301–1330. URL: https://ideas.repec.org/a/mcb/jmoncb/v41y2009i7p1301-1330.html.

de Walque, Grégory, Juha Kilponen, Massimiliano Pisani, Sebastian Schmidt, Vesna Corbo, Tibor Hledik, Josef Hollmayr, Samuel Hurtado, Paulo Júlio, Dmitry Kulikov, Matthieu Lemoine, and Matija Lo (2015). *Comparing Fiscal Multipliers Across Models and Countries in Europe*.

Working Paper Research 278. National Bank of Belgium. URL: https://ideas.repec.org/p/nbb/reswpp/201503-278.html.
Dean, Richard (2011). “Insurer Solvency Standards Reducing Risk in a Risk Business”. In: *Reserve Bank of New Zealand Bulletin* 74, pp. 1–6. URL: https://ideas.repec.org/a/nzb/nzbbul/dec20112.html.
Doepke, Matthias, Andreas Lehnert, and Andrew Sellgren (1999). “Macroeconomics”. Unpublished Textbook. URL: http://faculty.wcas.northwestern.edu/~mdo738/textbook/main.pdf.
ECRC (2018). *Recent Trends in and Preparedness for Extreme Weather Events*. Australian Senate Committee Report, Environment and Communications References Comittee. URL: https://www.aph.gov.au/Parliamentary_Business/Committees/Senate/Environment_and_Communications/Completed_inquiries/2010-13/extremeweather/report/index.
Eichengreen, Barry J., Asmaa A El-Ganainy, Kris James Mitchener, and Rui Pedro Esteves (2019). *Public Debt Through the Ages*. IMF Working Papers 2019/006. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2019-006.html.
Eo, Yunjong and James Morley (2017). *Why has the US Economy Stagnated Since the Great Recession?* Working Papers 2017-14. University of Sydney, School of Economics. URL: https://ideas.repec.org/p/syd/wpaper/2017-14.html.
EQC (2020). *Earthquake Commission Annual Report 2019/20*. URL: https://www.eqc.govt.nz/sites/public_files/documents/publications/EQC-Annual-Report-2019-20.pdf.
Erceg, Christopher and Jesper Lindé (2013). “Fiscal Consolidation in a Currency Union: Spending Cuts vs. Tax Hikes”. In: *Journal of Economic Dynamics and Control* 37.2, pp. 422–445. URL: https://EconPapers.repec.org/RePEc:eee:dyncon:v:37:y:2013:i:2:p:422-445.
Fair, Ray C. and John B. Taylor (1983). “Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models”. In: *Econometrica* 51.4, pp. 1169–1185. URL: https://ideas.repec.org/a/ecm/emetrp/v51y1983i4p1169-85.html.
Fatás, Antonio and Lawrence H. Summers (2018). “The Permanent Effects of Fiscal Consolidations”. In: *Journal of International Economics* 112.C, pp. 238–250. URL: https://ideas.repec.org/a/eee/inecon/v112y2018icp238-250.html.
Fernández-Villaverde, Jesús (2010). “The Econometrics of DSGE Models”. In: *SERIEs: Journal of the Spanish Economic Association* 1.1, pp. 3–49. URL: https://ideas.repec.org/a/spr/series/v1y2010i1p3-49.html.
Fookes, Craig (2011). *Modelling Shocks to New Zealand’s Fiscal Position*. Treasury Working Paper Series 11/02. New Zealand Treasury. URL: https://ideas.repec.org/p/nzt/nztwps/11-02.html.
Fornero, Jorge and Markus Kirchner (2018). “Learning about Commodity Cycles and Saving-Investment Dynamics in a Commodity-Exporting Economy”. In: *International Journal of Central Banking* 14.2, pp. 205–262. URL: https://ideas.repec.org/a/ijc/ijcjou/y2018q1a5.html.
Frame, Dave, Suzanne Rosier, Trevor Carey-Smith, Luke Harrington, Sam Dean, and Ilan Noy (2018). *Estimating Financial Costs of Climate Change in New Zealand*. New Zealand Climate Change Research Institute and NIWA Report. URL: https://treasury.govt.nz/sites/default/files/2018-08/LSF-estimating-financial-cost-of-climate-change-in-nz.pdf.
Furlanetto, Francesco, Antoine Lepetit, Ørjan Robstad, Juan Rubio-Ramrez, and Pål Ulvedal (2021). *Estimating Hysteresis Effects*. Working Papers 2021-11. FEDEA. URL: https://EconPapers.repec.org/RePEc:fda:fdaddt:2021-11.
Galì, Jordi and Tommaso Monacelli (2005). “Monetary Policy and Exchange Rate Volatility in a Small Open Economy”. In: *Review of Economic Studies* 72.3, pp. 707–734. URL: https://ideas.repec.org/a/oup/restud/v72y2005i3p707-734.html.
Galì, Jordi and Tommaso Monacelli (2008). “Optimal Monetary and Fiscal Policy in a Currency Union”. In: *Journal of International Economics* 76.1, pp. 116–132. URL: https://ideas.repec.org/a/eee/inecon/v76y2008i1p116-132.html.
Gallic, Ewen and Gauthier Vermandel (2019). *Weather Shocks*. Working Papers halshs-02127846. HAL. URL: https://ideas.repec.org/p/hal/wpaper/halshs-02127846.html.

Gemmell, Norman, Richard Kneller, and Ismael Sanz (2011). "The Timing and Persistence of Fiscal Policy Impacts on Growth: Evidence from OECD Countries". In: *Economic Journal* 121.550, pp. 33–58. URL: https://ideas.repec.org/a/ecj/econjl/v121y2011i550pf33-f58.html.

— (2014). "The Growth Effects of Tax Rates in the OECD". In: *Canadian Journal of Economics* 47.4, pp. 1217–1255. URL: https://ideas.repec.org/a/cje/issued/v47y2014i4p1217-1255.html.

Gertler, Mark (1999). "Government Debt and Social Security in a Life-Cycle Economy". In: *Carnegie-Rochester Conference Series on Public Policy* 50.1, pp. 61–110. URL: https://ideas.repec.org/a/eee/crcspp/v50y1999ip61-110.html.

Ghironi, Fabio (2017). "Policy Packages: Challenge and Opportunity for DSGE Research". In: *DSGE Models in the Conduct of Policy: Use as intended*. Ed. by Refet Gürkaynak and Cédric Tille. VOX CEPR Policy Portal. Chap. 6, pp. 61 –69.

Gomme, Paul and Peter Rupert (2007). "Theory, Measurement and Calibration of Macroeconomic Models". In: *Journal of Monetary Economics* 54.2, pp. 460–497. URL: https://ideas.repec.org/a/eee/moneco/v54y2007i2p460-497.html.

Gourio, Francois (2012). "Disaster Risk and Business Cycles". In: *American Economic Review* 102.6, pp. 2734–2766. URL: https://ideas.repec.org/a/aea/aecrev/v102y2012i6p2734-66.html.

Hall, Viv B. and C. John McDermott (2016). "Recessions and Recoveries in New Zealand's Post-Second World War Business Cycles". In: *New Zealand Economic Papers* 50.3, pp. 261–280. URL: https://ideas.repec.org/a/taf/nzecpp/v50y2016i3p261-280.html.

Heer, Burkhard and Alfred Maussner (2005). *Dynamic General Equilibrium Modelling: Computational Methods and Applications*. Springer Berlin Heidelberg. ISBN: 9783540220954. URL: https://books.google.co.nz/books?id=WLRZt6GnPn4C.

— (2009). *Dynamic General Equilibrium Modeling: Computational Methods and Applications*. Springer Berlin Heidelberg. ISBN: 9783540856856. URL: https://books.google.co.nz/books?id=ZdlEAAAAQBAJ.

Iacoviello, Matteo and Stefano Neri (2010). "Housing Market Spillovers: Evidence from an Estimated DSGE Model". In: *American Economic Journal: Macroeconomics* 2.2, pp. 125–164. URL: https://ideas.repec.org/a/aea/aejmac/v2y2010i2p125-64.html.

IAN (2019). *Fewer Cyclones Expected in Australia this Season*. Insurance Asia News Article. Accessed: 06-11-2019. URL: https://insuranceasianews.com/fewer-cyclones-expected-in-australia/.

ICNZ (2021). *Canterbury Earthquake Progress: Q1 2021*. Canterbury Earthquake Progress Statistics Q1 2021, Insurance Council of New Zealand. URL: https://www.icnz.org.nz/fileadmin/Assets/PDFs/Canterbury_Earthquake_Progress_Stats_Q1_2021.pdf.

IMF (2003). *New Zealand: 2003 Article IV Consultation–Staff Report; Staff Statement; Public Information Notice on the Executive Board Discussion; and Statement by the Executive Director for New Zealand*. International Monetary Fund. URL: https://books.google.co.nz/books?id=iNH9laGBjCwC.

Isoré, Marlène (2018). "Changes in Natural Disaster Risk: Macroeconomic Responses in Selected Latin American Countries". In: *Economies* 6.1, pp. 1–12. URL: https://ideas.repec.org/a/gam/jecomi/v6y2018i1p13-d133310.html.

Jones, Callum (2018). *Aging, Secular Stagnation and the Business Cycle*. IMF Working Papers 2018/067. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2018-067.html.

Judd, Kenneth (1998). *Numerical Methods in Economics*. MIT Press. URL: https://books.google.co.nz/books?id=9Wxk_z9HskAC.

Kaiser, Anna et al. (2012). "The Mw6.2 Christchurch Earthquake of February 2011: Preliminary Report". In: *New Zealand Journal of Geology and Geophysics* 55.1, 6790.

Kamber, Güneş, Chris McDonald, and Gael Price (2013). *Drying Out: Investigating the Economic Effects of Drought in New Zealand*. Reserve Bank of New Zealand Analytical Notes series AN2013/02. Reserve Bank of New Zealand. URL: https://ideas.repec.org/p/nzb/nzbans/2013-02.html.

Kamber, Güneş, Chris McDonald, Nicholas Sander, and Konstantinos Theodoridis (2015). *A Structural Model for Policy Analysis and Forecasting: NZSIM*. Reserve Bank of New Zealand Discussion Paper Series DP2015/05. Reserve Bank of New Zealand. URL: https://ideas.repec.org/p/nzb/nzbdps/2015-05.html.

Keen, Benjamin D. and Michael R. Pakko (2007). *Monetary Policy and Natural Disasters in a DSGE Model: How Should the Fed have Responded to Hurricane Katrina?* Tech. rep.

Kehoe, Timothy J. and Kim J. Ruhl (2003). *Recent Great Depressions: Aggregate Growth in New Zealand and Switzerland*. Tech. rep. URL: https://ideas.repec.org/p/cla/levarc/506439000000000529.html.

Kossin, James P., Suzana J. Camargo, and Matthew Sitkowski (2010). "Climate Modulation of North Atlantic Hurricane Tracks". In: *Journal of Climate* 23.11, pp. 3057–3076. URL: https://journals.ametsoc.org/view/journals/clim/23/11/2010jcli3497.1.xml.

Kulish, Mariano, Christopher Kent, and Kathryn Smith (2010). "Aging, Retirement, and Savings: A General Equilibrium Analysis". In: *The B.E. Journal of Macroeconomics* 10.1, pp. 1–32. URL: https://ideas.repec.org/a/bpj/bejmac/v10y2010i1n18.html.

Kumhof, Michael, Dirk V Muir, Susanna Mursula, and Douglas Laxton (2010). *The Global Integrated Monetary and Fiscal Model (GIMF) Theoretical Structure*. IMF Working Papers 2010/034. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfwpa/2010-034.html.

Leeper, Eric and Shu-Chun Yang (2008). "Dynamic Scoring: Alternative Financing Schemes". In: *Journal of Public Economics* 92.1-2, pp. 159–182. URL: https://EconPapers.repec.org/RePEc:eee:pubeco:v:92:y:2008:i:1-2:p:159-182.

Leeper, Eric M. (2010). *Monetary Science, Fiscal Alchemy*. NBER Working Papers 16510. National Bureau of Economic Research, Inc. URL: https://ideas.repec.org/p/nbr/nberwo/16510.html.

— (2011). "Anchors Aweigh: How Fiscal Policy Can Undermine Good Monetary Policy". In: *Monetary Policy under Financial Turbulence*. Ed. by Luis Felipe Cspedes, Roberto Chang, and Diego Saravia. Vol. 16. Central Banking, Analysis, and Economic Policies Book Series. Central Bank of Chile. Chap. 11, pp. 411–453. URL: https://ideas.repec.org/h/chb/bcchsb/v16c11pp411-453.html.

Leeper, Eric M., Michael Plante, and Nora Traum (2010). "Dynamics of Fiscal Financing in the United States". In: *Journal of Econometrics* 156.2, pp. 304–321. URL: https://ideas.repec.org/a/eee/econom/v156y2010i2p304-321.html.

Leeper, Eric M., Todd B. Walker, and Shu-Chun S. Yang (2010). "Government Investment and Fiscal stimulus". In: *Journal of Monetary Economics* 57.8, pp. 1000–1012. URL: https://ideas.repec.org/a/eee/moneco/v57y2010i8p1000-1012.html.

Lis, Eliza, Christiane Nickel, and Andrea Papetti (2020). *Demographics and Inflation in the Euro Area: a Two-Sector New Keynesian Perspective*. Working Paper Series 2382. European Central Bank. URL: https://EconPapers.repec.org/RePEc:ecb:ecbwps:20202382.

Liu, Philip (2006). *A Small New Keynesian Model of the New Zealand Economy*. Reserve Bank of New Zealand Discussion Paper Series DP2006/03. Reserve Bank of New Zealand. URL: https://ideas.repec.org/p/nzb/nzbdps/2006-03.html.

Liu, Xianglong, Adrian R. Pagan, and Tim Robinson (2018). "Critically Assessing Estimated DSGE Models: A Case Study of a Multisector Model". In: *The Economic Record* 94.307, pp. 349–371. URL: https://ideas.repec.org/a/bla/ecorec/v94y2018i307p349-371.html.

Mankiw, N. Gregory and Matthew Weinzierl (2006). "Dynamic Scoring: A Back-of-the-Envelope Guide". In: *Journal of Public Economics* 90.8-9, pp. 1415–1433. URL: https://ideas.repec.org/a/eee/pubeco/v90y2006i8-9p1415-1433.html.

Marcet, Albert and Juan Pablo Nicolini (1997). *Recurrent Hyperinflations and Learning*. Tech. rep.

MfE (2018). *Climate Change Projections for New Zealand: Atmospheric Projections Based on Simulations Undertaken for the IPCC 5th Assessment 2nd Edition*. Ministry for the

Environment Report. URL: https://www.mfe.govt.nz/sites/default/files/media/Climate%20Change/Climate-change-projections-2nd-edition-final.pdf.

MPI (2013). *The 2012-13 Drought: an Assessment and Historical Perspective; MPI Technical Paper No: 2012/18; Prepared for the Ministry for Primary Industries by NIWA*. URL: {https://niwa.co.nz/sites/niwa.co.nz/files/2013-18-The%202012-13%20drought%20an%20assessment%20and%20historical%20perspective.pdf}.

NIWA (2011). *Scenarios of Regional Drought under Climate Change*. NIWA Client Report. URL: https://www.niwa.co.nz/sites/niwa.co.nz/files/slmacc_drought_sldr093_june2011.pdf.

— (2019a). *Coastal Flooding Exposure Under Future Sea-level Rise for New Zealand*. NIWA Client Report. URL: https://www.deepsouthchallenge.co.nz/sites/default/files/2019-08/2019119WN_DEPSI18301_Coast_Flood_Exp_under_Fut_Sealevel_rise_FINAL%20%281%29_0.pdf.

— (2019b). *New Zealand Fluvial and Pluvial Flood Exposure*. NIWA Client Report. URL: https://www.deepsouthchallenge.co.nz/sites/default/files/2019-08/2019118WN_DEPSI18301_Flood%20Exposure_Final%20%281%29.pdf.

NRC (2010). *Improving Health Care Cost Projections for the Medicare Population: Summary of a Workshop. Committee on National Statistics, Division of Behavioral and Social Sciences and Education*. National Academies Press. ISBN: 9780309159760. URL: https://books.google.com.au/books?id=YoheAgAAQBAJ.

OAG (2017). *Commentary on He Tirohanga Mokopuna: 2016 Statement on the Long-term Fiscal Position*. Parliamentary papers presented to the House of Representatives of New Zealand by the Office of the Auditor-General. URL: https://www.oag.govt.nz/2017/long-term-fiscal-position/docs/long-term-fiscal-position.pdf.

OECD (2000). *OECD Economic Surveys: New Zealand 2000*. OECD Economic Surveys: New Zealand. OECD Publishing. ISBN: 9789264175860. URL: https://books.google.com.sv/books?id=0bjKK7pWrUQC.

— (2010). *Tax Policy Reform and Economic Growth*, p. 154. URL: https://www.oecd-ilibrary.org/content/publication/9789264091085-en.

— (2021). *Taxing Wages - New Zealand*. OECD. URL: https://www.oecd.org/tax/tax-policy/taxing-wages-new-zealand.pdf.

Ostry, Jonathan David, Atish R. Ghosh, and Raphael A Espinoza (2015). *When Should Public Debt Be Reduced?* IMF Staff Discussion Notes 2015/010. International Monetary Fund. URL: https://ideas.repec.org/p/imf/imfsdn/2015-010.html.

Pagan, Adrian (2005). *Some Econometric Analysis Of Constructed Binary Time Series*. CAMA Working Papers 2005-07. Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University. URL: https://ideas.repec.org/p/een/camaaa/2005-07.html.

Papetti, Andrea (2019). *Demographics and the Natural Real Interest Rate: Historical and Projected Paths for the Euro Area*. Working Paper Series 2258. European Central Bank. URL: https://ideas.repec.org/p/ecb/ecbwps/20192258.html.

Price, Robert, Thai-Thanh Dang, and Jarmila Botev (2015). *Adjusting Fiscal Balances for the Business Cycle: New Tax and Expenditure Elasticity Estimates for OECD Countries*. OECD Economics Department Working Papers 1275. OECD Publishing. URL: https://ideas.repec.org/p/oec/ecoaaa/1275-en.html.

QEAS (2017). *Economic Lessons from Queensland's Severe Tropical Cyclones*. Queensland Economic Advocacy Solutions Blog Entry.

Reddell, Michael and Cath Sleeman (2008). "Some Perspectives on Past Recessions". In: *Reserve Bank of New Zealand Bulletin* 71. URL: https://ideas.repec.org/a/nzb/nzbbul/june20082.html.

Santacreu, Ana Maria (2005). *Reaction Functions in a Small Open Economy: What Role for Non-Traded Inflation?* Reserve Bank of New Zealand Discussion Paper Series DP2005/04. Reserve Bank of New Zealand. URL: https://ideas.repec.org/p/nzb/nzbdps/2005-04.html.

Schmitt-Grohe, Stephanie and Martin Uribe (1997). “Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability”. In: *Journal of Political Economy* 105.5, pp. 976–1000. URL: https://ideas.repec.org/a/ucp/jpolec/v105y1997i5p976-1000.html.

— (2003). “Closing Small Open Economy Models”. In: *Journal of International Economics* 61.1, pp. 163–185. URL: https://ideas.repec.org/a/eee/inecon/v61y2003i1p163-185.html.

Sims, Eric and Jonathan Wolff (2018). “The Output And Welfare Effects Of Government Spending Shocks Over The Business Cycle”. In: *International Economic Review* 59.3, pp. 1403–1435. URL: https://ideas.repec.org/a/wly/iecrev/v59y2018i3p1403-1435.html.

Smets, Frank and Raf Wouters (2003). “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”. In: *Journal of the European Economic Association* 1.5, pp. 1123–1175. URL: https://ideas.repec.org/a/tpr/jeurec/v1y2003i5p1123-1175.html.

Smith, Christie and Christoph Thoenissen (2018). *Migration and Business Cycle Dynamics*. Reserve Bank of New Zealand Discussion Paper Series DP2018/07. Reserve Bank of New Zealand. URL: https://ideas.repec.org/p/nzb/nzbdps/2018-7.html.

Ter-Minassian, Teresa (2014). *External Review of the Treasury’s Policy Advice*. Tech. rep. International Monetary Fund. URL: https://treasury.govt.nz/sites/default/files/2014-10/tfpa-2908566.pdf.

The New Zealand Treasury (2014). *Canterbury Earthquake Fiscal Update*. Treasury Report. The New Zealand Treasury. URL: https://treasury.govt.nz/sites/default/files/2018-02/b14-2913046.pdf.

Trabandt, Mathias and Harald Uhlig (2011). “The Laffer Curve Revisited”. In: *Journal of Monetary Economics* 58.4, pp. 305–327. URL: https://EconPapers.repec.org/RePEc:eee:moneco:v:58:y:2011:i:4:p:305-327.

Traum, Nora and ShuChun S. Yang (2015). “When Does Government Debt Crowd Out Investment?” In: *Journal of Applied Econometrics* 30.1, pp. 24–45. URL: https://ideas.repec.org/a/wly/japmet/v30y2015i1p24-45.html.

Turnovsky, Stephen J. (2004). “The Transitional Dynamics of Fiscal Policy: Long-Run Capital Accumulation and Growth”. In: *Journal of Money, Credit and Banking* 36.5, pp. 883–910. URL: https://ideas.repec.org/a/mcb/jmoncb/v36y2004i5p883-910.html.

Turnovsky, Stephen J. and Santanu Chatterjee (2002). “To Spend the U.S. Government Surplus or to Increase the Deficit? A Numerical Analysis of the Policy Options”. In: *Journal of the Japanese and International Economies* 16.4, pp. 405–435. URL: https://ideas.repec.org/a/eee/jjieco/v16y2002i4p405-435.html.

Warne, Anders, Günter Coenen, and Kai Christoffel (2008). *The New Area-Wide Model of the Euro Area: a Micro-Founded Open-Economy Model for Forecasting and Policy Analysis*. Working Paper Series 944. European Central Bank. URL: https://ideas.repec.org/p/ecb/ecbwps/2008944.html.

Wood, Amy, Ilan Noy, and Miles Parker (2016). “The Canterbury Rebuild Five Years on from the Christchurch Earthquake”. In: *Reserve Bank of New Zealand Bulletin* 79, pp. 1–16. URL: https://ideas.repec.org/a/nzb/nzbbul/feb201603.html.

Woodford, Michael (2013). “Macroeconomic Analysis Without the Rational Expectations Hypothesis”. In: *Annual Review of Economics* 5.1, pp. 303–346. URL: https://ideas.repec.org/a/anr/reveco/v5y2013p303-346.html.

Wright, Allan and Patrice Borda (2016). *Macroeconomic Fluctuations Under Natural Disaster Shocks in Central America and the Caribbean*. IDB Publications (Working Papers) 8039. Inter-American Development Bank. URL: https://ideas.repec.org/p/idb/brikps/8039.html.

A. The Model

In this appendix I describe the stochastic neoclassical growth model (the NCGM) used in this paper. The model economy is populated by a representative household, a perfectly competitive intermediate goods producer, a perfectly competitive final goods producer, a fiscal authority and a foreign economy. Section 2 explains the production structure (Figure 1) and the relationship between the different agents and sectors (Figure 2) diagramatically.

A.1 The Representative Household

The representative household derives positive utility from effective consumption relative to last period's aggregate effective consumption and the current level of labour-augmenting technology. They derive negative utility from working. Following Barro (1980) I introduce more flexibility into the model by allowing for the degree of substitutability or complementarity between public and private consumption to be fine tuned. The representative household's period utility function, $u\left(c_t, \ell_t\right)$, is given by,

$$u\left(c_t, \ell_t\right) = \frac{\mathscr{A}_{c,t}\left(c_t^{\star} - \chi\left(1+\gamma_t\right)\bar{c}_{t-1}^{\star}\right)^{1-\sigma} A_t^{\sigma-1}}{1-\sigma} - \left(1-\chi\right)^{-\sigma} \varkappa_t \kappa_t h_t \frac{l_t^{1+\eta}}{1+\eta}, \tag{1}$$

$$c_t^{\star} = c_t + \alpha_{g_g} g_{g,t} + \alpha_{g_i} g_{i,t} + \alpha_{g_h} g_{h,t}, \tag{2}$$

where $\mathscr{A}_{c,t}$ is an exogenous consumption preference shock process, $c_t^{\star}$ is effective consumption per capita, c_t is consumption per capita, $g_{g,t}$ is per capita general government expenditure, $g_{i,t}$ is per capita government investment, $g_{h,t}$ is per capita health expenditure, A_t is the current level of labour-augmenting technology, γ_t is the growth rate of labour-augmenting technology and l_t is hours worked. The bar over per capita consumption indicates habit formation is external. I let σ denote the inverse of the intertemporal elasticity of substitution, χ the weight on habit formation, η is the inverse of the Frisch elasticity of labour supply and α_{g_g}, α_{g_i} and α_{g_h} determine the degree of substitution or complementarity between private consumption and the components of government expenditure. Following Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020), I introduce exogenously determined demographic wedges for the aggregate disutility of working, $\varkappa_t$ and human capital, h_t. Expressions for these are derived in Appendix E. I also introduce κ_t, the short-run time-varying weight on the disutility of working, which follows the exogenous process,

$$\log \kappa_t = \rho_\kappa \log \kappa_{t-1} + \left(1-\rho_\kappa\right)\log\kappa + \varepsilon_{\kappa,t}, \tag{3}$$

with ρ_κ governing the persistence of the process and $\varepsilon_{\kappa,t}$ representing an exogenous leisure preference shock.

The representative household maximises the expected present value of their lifetime utility subject to the budget constraint and the law of motion for physical capital. The aggregate budget constraint is given by,

$$\begin{aligned}
&\left(1+\tau_t^c\right)C_t + I_t + B_t\exp\left(\mathscr{A}_{b,t}\right) + F_t\exp\left(\mathscr{A}_{b,t}\right) = \ldots \\
&\quad \ldots\left(1+r_{t-1}\right)B_{t-1} + \left(1+r_{t-1}\right)F_{t-1} + \left(1-\tau_t^k\right)R_t\upsilon_t\left(1-\varphi\Delta_t\right)\bar{K}_{t-1} - \ldots \\
&\quad \ldots - \left(\gamma_{\upsilon,1}\left(\upsilon_1 - 1\right) + \frac{\gamma_{\upsilon,2}}{2}\left(\upsilon_t-1\right)^2\right)\left(1-\varphi\Delta_t\right)\bar{K}_{t-1} + \left(1-\tau_t^l\right)w_t\ell_t + \ldots \\
&\quad + \ldots\omega\tau_t^k\delta\left(1-\varphi\Delta_t\right)\bar{K}_{t-1} + \left(1-\tau^s\right)w_t dN_{r,t} - Z_t + \mathfrak{E}_t + \Xi_t,
\end{aligned} \tag{4}$$

where C_t is aggregate consumption, I_t is aggregate investment, B_t is aggregate net government debt, F_t is aggregate net foreign debt denominated in domestic currency, $\mathscr{A}_{b,t}$ is an autoregressive shock process that is equivalent to a risk premium shock, r_t is the real interest rate, R_t is the gross dividend or rental rate on private capital, υ_t is the (variable) capacity utilisation rate, $\bar{K}_t$

is the stock of physical capital, w_t is the real wage level, N_t is the total population over the age of 15, Z_t is net lump-sum taxes (transfers) paid by (to) the household, $\mathfrak{E}_t$ is payments made by the Earthquake Commission (EQC) and Ξ_t is adjustment costs, which are rebated to the household.

I introduce $N_{r,t} = \sum_{j=jr}^{J} N_{t,j}$ to capture the total number of people eligible for superannuation at time t, where $N_{t,j}$ is the number of people of age j alive at time t, jr is the age of eligibility for superannuation, which is 65 in this case, and J is the maximum life expectancy, which is 95+ in this case, so that total real superannuation payments, $w_t d N_{r,t}$, are a function of the total population over the age of 65 (those eligible for superannuation payments) and are indexed to the real wage.[72]

I let τ_t^c denote the consumption tax rate, τ_t^k is the average tax rate on dividend and other capital income and τ_t^l is the average tax rate on labour income. The fraction of depreciation deducted from the tax bill on capital income is denoted by ω, with δ representing the depreciation rate on physical capital.

The representative household is subject to a quadratic adjustment cost, $\left(\gamma_{\upsilon,1}\left(\upsilon_t - 1\right) + \frac{\gamma_{\upsilon,2}}{2}\left(\upsilon_t - 1\right)^2\right)\left(1 - \varphi\Delta_t\right)\bar{K}_{t-1}$, when determining the utilisation rate on physical capital, where $\gamma_{\upsilon,2}$ determines how costly it is to adjust capacity utilisation. This is the same specification used by Warne, Coenen, and Christoffel (2008). I introduce Δ_t, an exogenous indicator variable that takes the value 0 in normal times and 1 in a period when a natural disaster, like an earthquake or a storm, hits the economy. The parameter φ determines the fraction of the capital stock owned by the household that is destroyed by the natural disaster. This assumption is used by Fornero and Kirchner (2018) to model earthquakes in Chile and by Wright and Borda (2016) to model natural disasters (hurricanes, storms, earthquakes) more generally in the Carribean and Central America.[73]

Physical capital evolves according to the aggregate law of motion,

$$\bar{K}_t = \left[1 - \frac{\phi}{2}\left(\frac{\mathscr{A}_{i,t} I_t}{I_{t-1}} - \mu\right)^2\right] I_t + \left(1 - \varphi\Delta_t\right)\left(1 - \delta\right)\bar{K}_{t-1}, \tag{5}$$

where $\mu = (1+\gamma)(1+n)$ is the steady state aggregate growth rate, γ is the steady state labour-augmenting total factor productivity growth rate, n is the steady state population growth rate, ϕ determines how costly it is to adjust investment and $\mathscr{A}_{i,t}$ is the autoregressive shock process for investment efficiency shocks. This form of investment adjustment costs has been used by Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005) among others.

The representative household maximises the expected discounted sum of current and future utilities by choosing per capita allocations of consumption, $c_t = \frac{C_t}{N_t}$, investment $i_t = \frac{I_t}{N_t}$, government debt $b_t = \frac{B_t}{N_t}$, net foreign debt $f_t = \frac{F_t}{N_t}$, physical capital $\bar{k}_t = \frac{\bar{K}_t}{N_t}$, hours worked l_t and capacity utilisation υ_t, where the working age population (the population over the age of 15) grows according to,

$$N_t = \left(1 + n_t\right) N_{t-1}. \tag{6}$$

72 Statistics New Zealand population projections have yearly age categories from ages 0 through to 94, and a catchall category of 95+ for those aged 95 years and older. Total population aged 15 and over would be equivalend to $N_t = \sum_{j=15}^{J} N_{t,j}$.

73 A similar approach has been used by Isoré (2018) to model natural disaster risk in selected Latin American countries and by Keen and Pakko (2007) to model the Federal Reserve's response to Hurricane Katrina.

Rewriting the aggregate budget constraint (4) in per capita terms yields,

$$
\begin{aligned}
&(1+\tau_t^c)\,c_t + i_t + b_t \exp(\mathscr{A}_{b,t}) + f_t \exp(\mathscr{A}_{b,t}) = \ldots \\
&\ldots (1+r_{t-1})\,b_{t-1}(1+n_t)^{-1} + (1+r_{t-1})\,f_{t-1}(1+n_t)^{-1} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} - \ldots \\
&\ldots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} + \left(1-\tau_t^l\right) w_t h_t l_t + \ldots \\
&+ \ldots \omega\tau_t^k \delta (1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} + (1-\tau^s)\,w_t d\Psi_t + \mathfrak{e}_t - z_t + \xi_t \qquad (7)
\end{aligned}
$$

where $z_t = \frac{Z_t}{N_t}$ is total net lump-sum taxes (transfers) in per capita terms, $\mathfrak{e}_t = \frac{\mathfrak{E}_t}{N_t}$ is per capita EQC payments, $\xi_t = \frac{\Xi_t}{N_t}$ is adjustment costs in per capita terms, $w_t d\Psi_t$ is superannuation payments in per capita terms and Ψ_t, is a measure of the old-age dependency ratio, defined as

$$
\Psi_t = \frac{N_{r,t}}{N_t} = \frac{\sum_{j=jr}^{J} N_{t,j}}{N_t}. \qquad (8)
$$

The law of motion for physical capital (5) can also be rewritten in per capita terms as follows,

$$
\bar{k}_t = \left[1 - \frac{\phi}{2}\left(\frac{\mathscr{A}_{i,t} i_t}{i_{t-1}}(1+n_t) - \mu\right)^2\right] i_t + (1-\varphi\Delta_t)(1-\delta)\,\bar{k}_{t-1}(1+n_t)^{-1}. \qquad (9)
$$

Setting up the Lagrangean for the representative household,

$$
\mathscr{L}_0 = E_0 \left\{ \sum_{t=0}^{\infty} N_t \beta^t \left[
\begin{array}{c}
\frac{\mathscr{A}_{c,t}\left(c_t^\star - \chi(1+\gamma_t)\,c_{t-1}^\star\right)^{1-\sigma} A_t^{\sigma-1}}{1-\sigma} - (1-\chi)^{-\sigma} \varkappa_t \kappa_t h_t \frac{l_t^{1+\eta}}{1+\eta} - \ldots \\
\ldots - \lambda_t \left[
\begin{array}{c}
(1+\tau_t^c)\,c_t + i_t + b_t \exp(\mathscr{A}_{b,t}) + f_t \exp(\mathscr{A}_{b,t}) - \ldots \\
\ldots - (1+r_{t-1}) b_{t-1}(1+n_t)^{-1} - (1+r_{t-1}) f_{t-1}(1+n_t)^{-1} - \ldots \\
\ldots - (1-\tau_t^k)\,R_t \upsilon_t (1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} + \ldots \\
\ldots + \left(\gamma_{\upsilon,1}(\upsilon_t - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t-1)^2\right)(1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} - \ldots \\
\ldots - (1-\tau_t^l)\,w_t h_t l_t - \omega\tau^k \delta (1-\varphi\Delta_t)\,\bar{k}_{t-1}(1+n_t)^{-1} - \ldots \\
\ldots - (1-\tau^s)\,w_t d\Psi_t + z_t - \mathfrak{e}_t - \xi_t + \ldots \\
\ldots + \Phi_t \left[
\begin{array}{c}
\bar{k}_t - \left[1 - \frac{\phi}{2}\left(\frac{\mathscr{A}_{i,t} i_t}{i_{t-1}}(1+n_t) - \mu\right)^2\right] i_t - \ldots \\
\ldots - (1-\varphi\Delta_t)(1-\delta)\,\bar{k}_{t-1}(1+n_t)^{-1}
\end{array}
\right]
\end{array}
\right]
\end{array}
\right] \right\}, \qquad (10)
$$

where λ_t is the shadow price of income/wealth, Φ_t is the shadow price of installed capital (Tobin's Q) and β is the household's discount factor. I obtain the representative household's first order conditions, for consumption,

$$
\frac{\partial \mathscr{L}_t}{\partial c_t} = \mathscr{A}_{c,t}\left(c_t^\star - \chi(1+\gamma_t)\,c_{t-1}^\star\right)^{-\sigma} A_t^{\sigma-1} - \lambda_t (1+\tau_t^c) = 0, \qquad (11)
$$

for hours worked,

$$
\frac{\partial \mathscr{L}_t}{\partial \ell_t} = -(1-\chi)^{-\sigma} \varkappa_t \kappa_t h_t l_t^\eta + \lambda_t \left(1-\tau_t^l\right) w_t h_t = 0, \qquad (12)
$$

for net government debt,

$$
\frac{\partial \mathscr{L}_t}{\partial b_t} = -N_t \lambda_t \exp(\mathscr{A}_{b,t}) + E_t\left\{N_{t+1}\beta\lambda_{t+1}(1+r_t)(1+n_{t+1})^{-1}\right\} = 0, \qquad (13)
$$

for net foreign debt,

$$
\frac{\partial \mathscr{L}_t}{\partial f_t} = -N_t \lambda_t \exp(\mathscr{A}_{b,t}) + E_t\left\{N_{t+1}\beta\lambda_{t+1}(1+r_t)(1+n_{t+1})^{-1}\right\} = 0, \qquad (14)
$$

for capital,

$$\frac{\partial \mathscr{L}_t}{\partial \bar{k}_t} = -N_t\lambda_t\Phi_t + \dots$$

$$\dots + E_t\left\{\begin{array}{c} N_{t+1}\beta\lambda_{t+1}\left(1+n_{t+1}\right)^{-1}\left(1-\varphi\Delta_{t+1}\right)\times\dots \\ \dots\times\left[\begin{array}{c}\left(1-\tau^k_{t+1}\right)R_{t+1}\upsilon_{t+1}-\gamma_{\upsilon,1}\left(\upsilon_{t+1}-1\right)-\frac{\gamma_{\upsilon,2}}{2}\left(\upsilon_{t+1}-1\right)^2+\dots \\ \dots+\omega\tau^k_{t+1}\delta+\Phi_{t+1}\left(1-\delta\right)\end{array}\right]\end{array}\right\} = 0, \quad (15)$$

for investment,

$$\frac{\partial \mathscr{L}_t}{\partial i_t} = -N_t\lambda_t + N_t\lambda_t\Phi_t\left[\begin{array}{c} 1-\frac{\phi}{2}\left(\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)-\mu\right)^2-\dots \\ \dots-\phi\left(\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)-\mu\right)\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)\end{array}\right]+\dots$$

$$\dots + E_t\left\{N_{t+1}\beta\lambda_{t+1}\Phi_{t+1}\phi\left(\frac{\mathscr{A}_{i,t+1}i_{t+1}}{i_t}\left(1+n_{t+1}\right)-\mu\right)\left(\frac{\mathscr{A}_{i,t+1}i_{t+1}}{i_t}\right)^2\left(1+n_{t+1}\right)\right\} = 0, \quad (16)$$

and for capacity utilisation,

$$\frac{\partial \mathscr{L}_t}{\partial \upsilon_t} = -\lambda_t\left[\begin{array}{c}\left(\gamma_{\upsilon,1}+\gamma_{\upsilon,2}\left(\upsilon_t-1\right)\right)\left(1-\varphi\Delta_t\right)\bar{k}_{t-1}\left(1+n_t\right)^{-1}-\dots \\ \dots-\left(1-\tau^k_t\right)R_t\left(1-\varphi\Delta_t\right)\bar{k}_{t-1}\left(1+n_t\right)^{-1}\end{array}\right] = 0. \quad (17)$$

From (13) I get the Euler equation,

$$\lambda_t \exp\left(\mathscr{A}_{b,t}\right) = E_t\left\{\beta\lambda_{t+1}\left(1+r_t\right)\right\}. \quad (18)$$

From (11) I get the marginal utility of consumption,

$$\lambda_t = \frac{\mathscr{A}_{c,t}\left(c^\star_t-\chi\left(1+\gamma_t\right)c^\star_{t-1}\right)^{-\sigma}A_t^{\sigma-1}}{1+\tau^c_t}. \quad (19)$$

From (12) I get the marginal rate of substitution,

$$\lambda_t w_t = \left(1-\chi\right)^{-\sigma}\varkappa_t\kappa_t\left(\frac{l^\eta_t}{1-\tau^l_t}\right). \quad (20)$$

From (15) I get the asset pricing equation for Tobin's Q,

$$\Phi_t = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-\varphi\Delta_{t+1}\right)\left[\begin{array}{c}\left(1-\tau^k_{t+1}\right)R_{t+1}\upsilon_{t+1}-\gamma_{\upsilon,1}\left(\upsilon_{t+1}-1\right)-\dots \\ \dots-\frac{\gamma_{\upsilon,2}}{2}\left(\upsilon_{t+1}-1\right)^2+\omega\tau^k_{t+1}\delta+\Phi_{t+1}\left(1-\delta\right)\end{array}\right]\right\}. \quad (21)$$

From (16) I get,

$$1 = \Phi_t\left[1-\frac{\phi}{2}\left(\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)-\mu\right)^2-\phi\left(\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)-\mu\right)\frac{\mathscr{A}_{i,t}i_t}{i_{t-1}}\left(1+n_t\right)\right]+\dots$$

$$\dots+E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\Phi_{t+1}\phi\left(\frac{\mathscr{A}_{i,t+1}i_{t+1}}{i_t}\left(1+n_{t+1}\right)-\mu\right)\left(\frac{\mathscr{A}_{i,t+1}i_{t+1}}{i_t}\left(1+n_{t+1}\right)\right)^2\right\}. \quad (22)$$

And from (17) I equate the marginal benefit of increasing capacity utilisation with the marginal cost,

$$\left(1-\tau^k_t\right)R_t = \gamma_{\upsilon,1}+\gamma_{\upsilon,2}\left(\upsilon_t-1\right). \quad (23)$$

A.2 The Representative Intermediate Goods Producer

The representative intermediate goods producing firm produces intermediate goods according to the Cobb Douglas production technology,

$$Y_t = \mathscr{A}_t K_t^{\theta} \left(A_t \ell_t\right)^{1-\theta} \left(\left(1-\varphi_G \Delta_t\right) \frac{K_{G,t-1}}{A_t N_t}\right)^{\theta_G}, \tag{24}$$

where Y_t is intermediate output (gross domestic product), $K_t = \left(1-\varphi \Delta_t\right) \upsilon_t \bar{K}_{t-1}$ is the effective stock of physical capital, $K_{G,t-1}$ is the stock of government capital, φ_G is the proportion of the government's capital stock destroyed in a natural disaster, A_t is labour-augmenting technology and $\mathscr{A}_t$ is neutral technology. Capital's share of income θ, satisfies $0 < \theta < 1$ and $\theta_G \geq 0$ is the elasticity of output with respect to public capital.

This type of production function has been used by Leeper, Walker, and Yang (2010) and Traum and Yang (2015) to incorporate government investment and public capital into both the model and the production function. In my specification, public capital enters the production function relative to labour-augmenting technology and the population. This ensures the increasing returns to scale production function exhibits balanced growth.[74] It also means if investment in infrastructure does not keep pace with productivity and the population level, the infrastructure deficit will be a drag on the economy.

I adopt the following specification for labour-augmenting technology,

$$A_t = \left(1+\gamma_t\right) A_{t-1}, \tag{25}$$

where,

$$1+\gamma_t = \left(1+\gamma\right) \exp\left(\mathscr{A}_{\gamma,t} - \mathscr{A}_{\gamma,t-1}\right), \tag{26}$$

and,

$$\mathscr{A}_{\gamma,t} = \rho_{\mathscr{A}_\gamma} \mathscr{A}_{\gamma,t-1} + \varepsilon_{\gamma,t}. \tag{27}$$

$\mathscr{A}_{\gamma,t}$ is an exogenous shock process, $\varepsilon_{\gamma,t}$ is a labour-augmenting technology shock and $\rho_{\mathscr{A}_\gamma}$ governs the persistence of this process. This specification for technology, used by Iacoviello and Neri (2010), nests both deterministic ($0 \leq \rho_{\mathscr{A}_\gamma} < 1$) and unit root ($\rho_{\mathscr{A}_\gamma} = 1$) trend processes. Neutral technology evolves according to,

$$\log \mathscr{A}_t = \rho_{\mathscr{A}} \log \mathscr{A}_{t-1} + \varepsilon_{\mathscr{A},t}. \tag{28}$$

I can rewrite the production function (24) in per capita terms as follows,

$$Y_t = \mathscr{A}_t K_t^{\theta} \left(A_t N_t h_t l_t\right)^{1-\theta} \left(\left(1-\varphi_G \Delta_t\right) \frac{K_{G,t-1}}{A_t N_t}\right)^{\theta_G}, \tag{29}$$

$$y_t = \mathscr{A}_t k_t^{\theta} \left(A_t h_t l_t\right)^{1-\theta} \left(\left(1-\varphi_G \Delta_t\right) \frac{k_{G,t-1}}{A_t} \left(1+n_t\right)^{-1}\right)^{\theta_G}, \tag{30}$$

where $\ell_t = N_t h_t l_t$, $y_t = \frac{Y_t}{N_t}$, $k_t = \frac{K_t}{N_t} = \left(1-\varphi \Delta_t\right) \frac{\upsilon_t \bar{K}_{t-1}}{N_t} = \left(1-\varphi \Delta_t\right) \upsilon_t \bar{k}_{t-1} \left(1+n_t\right)^{-1}$ and $k_{G,t-1} = \frac{K_{G,t-1}}{N_{t-1}}$. The perfectly competitive intermediate goods producer makes the following real profits,

$$\mathscr{P}_t = \frac{p_{H,t}}{p_{c,t}} Y_t - R_t K_t - w_t N_t h_t l_t, \tag{31}$$

[74] Leeper, Walker, and Yang (2010) and Traum and Yang (2015) do not include trends in their models, which means they do not need to worry about balanced growth paths. In fact, my specification is equivalent to theirs when the model is written in detrended form.

where $p_{H,t}$ is the price of domestically produced intermediate goods and $p_{c,t}$ is the price of the final good, which I treat as the numeraire. In per capita terms,

$$\mathscr{p}_t = p_t y_t - R_t k_t - w_t h_t l_t, \tag{32}$$

where $p_t = \frac{p_{H,t}}{p_{c,t}}$ is the relative price of the GDP and final goods deflators. Substituting in the production function (30),

$$\mathscr{p}_t = p_t \mathscr{A}_t k_t^{\theta} \left(A_t h_t l_t\right)^{1-\theta} \left(\left(1 - \varphi_G \Delta_t\right) \frac{k_{G,t-1}}{A_t} \left(1 + n_t\right)^{-1} \right)^{\theta_G} - R_t k_t - w_t h_t l_t. \tag{33}$$

I obtain the first order conditions with respect to capital and labour,

$$\frac{\partial \mathscr{p}_t}{\partial k_t} = \theta p_t \mathscr{A}_t k_t^{\theta-1} \left(A_t h_t l_t\right)^{1-\theta} \left(\left(1 - \varphi_G \Delta_t\right) \frac{k_{G,t-1}}{A_t} \left(1 + n_t\right)^{-1} \right)^{\theta_G} - R_t = 0, \tag{34}$$

$$\frac{\partial \mathscr{p}_t}{\partial l_t} = \left(1-\theta\right) p_t \mathscr{A}_t k_t^{\theta} \left(A_t h_t\right)^{1-\theta} l_t^{-\theta} \left(\left(1 - \varphi_G \Delta_t\right) \frac{k_{G,t-1}}{A_t} \left(1 + n_t\right)^{-1} \right)^{\theta_G} - w_t h_t = 0, \tag{35}$$

From the first order condition on physical capital I obtain,

$$\theta p_t \frac{y_t}{k_t} = R_t, \tag{36}$$

so that the rental/dividend rate on physical capital is equal to the marginal product of capital. From the first order condition on hours worked, I obtain,

$$\left(1-\theta\right) p_t \frac{y_t}{h_t l_t} = w_t, \tag{37}$$

so that the real wage is equal to the marginal product of labour.

A.3 The Representative Final Goods Producer

Final goods are produced by combining domestically produced and imported intermediate goods according to the CES production technology,

$$\mathcal{A}_t = \left[\alpha^{\frac{1}{\varepsilon}} Y_{H,t}^{1-\frac{1}{\varepsilon}} + \left(1-\alpha\right)^{\frac{1}{\varepsilon}} Y_{F,t}^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{38}$$

where $\mathcal{A}_t$ is the aggregate final good, $Y_{H,t}$ is the domestically produced intermediate good, $Y_{F,t}$ is the imported intermediate good, ε is the elasticity of substitution between domestically produced and foreign intermediates and α is the home bias parameter. In per capita terms (38) becomes,

$$a_t = \left[\alpha^{\frac{1}{\varepsilon}} y_{H,t}^{1-\frac{1}{\varepsilon}} + \left(1-\alpha\right)^{\frac{1}{\varepsilon}} y_{F,t}^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{39}$$

where $a_t = \frac{\mathcal{A}_t}{N_t}$, $y_{H,t} = \frac{Y_{H,t}}{N_t}$, and $y_{F,t} = \frac{Y_{F,t}}{N_t}$. The final goods producer chooses allocations of domestic and foreign produced intermediates to minimise their costs. Setting up the final goods producer's Lagrangean gives,

$$\mathscr{L}_t = p_{H,t} y_{H,t} + \mathscr{s}_t p_t^* y_{F,t} - p_{c,t} \left(\left[\alpha^{\frac{1}{\varepsilon}} y_{H,t}^{1-\frac{1}{\varepsilon}} + \left(1-\alpha\right)^{\frac{1}{\varepsilon}} y_{F,t}^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} - a_t \right), \tag{40}$$

where $\mathscr{s}_t$ is the nominal exchange rate, representing the domestic currency price of one unit of foreign currency,[75] and p_t^* is the final goods deflator in the foreign country, denominated in

[75] This means that a decrease in the exchange rate is an appreciation and an increase in the exchange rate is a depreciation.

foreign currency. I assume that the domestic economy is too small to affect the foreign economy and that imports from the domestic economy make up a neglible contribution to the production of final goods in the foreign country. As a consequence the price deflator in the foreign intermediate and final goods producing sectors is assumed to be the same and $\mathfrak{z}_t p_t^* = p_{F,t}$ represents the price of imported intermediate goods in domestic currency. I obtain the first order conditions for domestically produced intermediate goods,

$$\frac{\partial \mathscr{L}_t}{\partial y_{H,t}} = p_{H,t} - p_{c,t} \alpha^{\frac{1}{\varepsilon}} y_{H,t}^{-\frac{1}{\varepsilon}} a_t^{\frac{1}{\varepsilon}}, \tag{41}$$

and foreign produced intermediate goods,

$$\frac{\partial \mathscr{L}_t}{\partial y_{F,t}} = \mathfrak{z}_t p_t^* - p_{c,t} \left(1-\alpha\right)^{\frac{1}{\varepsilon}} y_{F,t}^{-\frac{1}{\varepsilon}} a_t^{\frac{1}{\varepsilon}}. \tag{42}$$

From the first order conditions (41) and (42) I obtain the demand functions for home and foreign produced intermediate goods,

$$y_{H,t} = \alpha \left(\frac{p_{H,t}}{p_{c,t}}\right)^{-\varepsilon} a_t, \tag{43}$$

$$y_{F,t} = \left(1-\alpha\right) \left(\frac{\mathfrak{z}_t p_t^*}{p_{c,t}}\right)^{-\varepsilon} a_t. \tag{44}$$

By substituting (43) and (44) into (39) I obtain the price index for final goods,

$$p_{c,t} = \left[\alpha p_{H,t}^{1-\varepsilon} + \left(1-\alpha\right)\left(\mathfrak{z}_t p_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{45}$$

The terms of trade are defined as follows,

$$\Gamma_t = \frac{\mathfrak{z}_t p_t^*}{p_{H,t}} = \frac{p_{F,t}}{p_{H,t}}. \tag{46}$$

Substituting (46) into (45) allows me to write the relative price of the intermediate good as a function of the terms of trade,

$$p_t = \left[\alpha + \left(1-\alpha\right)\Gamma_t^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}}. \tag{47}$$

The real exchange rate is defined as,

$$q_t = \frac{\mathfrak{z}_t p_t^*}{p_{c,t}}. \tag{48}$$

Consistent with my definition of the nominal exchange rate, a fall in the real exchange rate represents a real appreciation while an increase represents a depreciation. My defintion of the real exchange rate implies,

$$q_t = \frac{\mathfrak{z}_t p_t^*}{\cancel{p_{H,t}}} \frac{\cancel{p_{H,t}}}{p_{c,t}} = \frac{\mathfrak{z}_t p_t^*}{p_{c,t}} = \Gamma_t p_t. \tag{49}$$

A.4 Government

Total government spending (G_t) is comprised of government spending on health ($G_{h,t}$), government investment ($G_{i,t}$) and general government expenditure ($G_{g,t}$), so that,

$$G_t = G_{h,t} + G_{i,t} + G_{g,t}, \tag{50}$$

which in per capita terms becomes,

$$g_t = g_{h,t} + g_{i,t} + g_{g,t}. \tag{51}$$

Taxes net of transfers and superannuation payments are defined as,

$$T_t = \tau_t^c C_t + \tau_t^l w_t \ell_t + \tau_t^k \left(R_t \upsilon_t - \omega\delta\right)\left(1-\varphi\Delta_t\right)\bar{K}_{t-1} - \left(1-\tau^s\right) w_t d N_{r,t} + Z_t - \mathfrak{E}_t. \tag{52}$$

where τ^s is the average tax rate on superannuation payments. In per capita terms this becomes,

$$t_t = \tau_t^c c_t + \tau_t^l w_t h_t l_t + \tau_t^k \left(R_t \upsilon_t - \omega\delta\right)\left(1-\varphi\Delta_t\right)\bar{k}_{t-1}\left(1+n_t\right)^{-1} - \left(1-\tau^s\right) w_t d\Psi_t + z_t - \mathfrak{e}_t, \tag{53}$$

The aggregate government budget constraint is given by,

$$B_t = G_t - T_t + \left(1+r_{t-1}\right) B_{t-1}, \tag{54}$$

which in per capita terms becomes,

$$b_t = g_t - t_t + \left(1+r_{t-1}\right) b_{t-1}\left(1+n_t\right)^{-1}. \tag{55}$$

The public capital stock evolves according to the law of motion,

$$K_{G,t} = \left(1-\varphi_G\Delta_t\right)\left(1-\delta_G\right) K_{G,t-1} + G_{i,t}, \tag{56}$$

which can be rewritten in per capita terms as follows,

$$k_{G,t} = \left(1-\varphi_G\Delta_t\right)\left(1-\delta_G\right) k_{G,t-1}\left(1+n_t\right)^{-1} + g_{i,t}. \tag{57}$$

The spending rules for government consumption and investment set government expenditure as a ratio of GDP. This implies,

$$g_{h,t} = \vartheta_{h,t} p_t y_t \mathcal{A}_{g_h,t}, \tag{58}$$

$$g_{i,t} = \vartheta_i p_t y_t \mathcal{A}_{g_i,t}, \tag{59}$$

$$g_{g,t} = \vartheta_{g,t} p_t y_t \mathcal{A}_{g_g,t}, \tag{60}$$

where $\vartheta_{h,t}$, ϑ_i and $\vartheta_{g,t}$ determine the ratio of government spending to GDP, and $\mathcal{A}_{g_h,t}$, $\mathcal{A}_{g_i,t}$ and $\mathcal{A}_{g_g,t}$ are autoregressive shock processes that allow temporary deviations from the respective spending rules. $\vartheta_{h,t}$ and $\vartheta_{g,t}$ have time subscripts indicating that they can vary over time. These will be set exogenously to take into account projected changes in health and other government spending. The capital, consumption and labour tax rates are determined by the following rules,

$$\log\left(\frac{\tau_t^k}{\tau_t^{k\star}}\right) = \psi_{\tau^k}\left(\frac{b_{t-1}}{4\times p_{t-1}y_{t-1}} - b_t^\star\right), \tag{61}$$

$$\log\left(\frac{\tau_t^c}{\tau_t^{c\star}}\right) = \psi_{\tau^c}\left(\frac{b_{t-1}}{4\times p_{t-1}y_{t-1}} - b_t^\star\right), \tag{62}$$

$$\log\left(\frac{\tau_t^l}{\tau_t^{l\star}}\right) = \psi_{\tau^l}\left(\frac{b_{t-1}}{4\times p_{t-1}y_{t-1}} - b_t^\star\right), \tag{63}$$

where ψ_j for $j = \tau^k, \tau^c, \tau^l$ are the response coefficients for the deviation of net debt to GDP from the target for each of the tax types, b_t^* is the potentially time-varying debt to GDP target, and τ_t^{j*} for $j = k, c, l$ is the potentially time-varying tax rate target. The net debt target is altered in the fiscal consolidation scenarios and the delayed response scenarios where net debt is stabilised at a permanently higher level. The target tax rates are gradually increased over the reporting period to prevent large jumps or changes in the tax rates required to approximately balance the budget. Net lump sum taxes (transfers) are set as a share of GDP according to the rule,

$$\tilde{z}_t = \vartheta_{z,t} p_t \tilde{y}_t \mathcal{A}_{z,t}, \tag{64}$$

in a similar fashion to government consumption and government investment, where $\vartheta_{z,t}$ is the potentially time-varying transfers payments to GDP ratio, which is set exogenously, and $\mathcal{A}_{z,t}$ is an autoregressive shock process that allows for temporary deviations from the rule. EQC payments are only made when there is an earthquake and there is not enough money in the disaster relief fund to pay for expenses. I treat them as cash transfers payments to the representative household, who owns the private capital stock. Because EQC payments are zero in normal times, I model per capita EQC payments as an autoregressive process centered on zero,

$$\tilde{\mathfrak{e}}_t = \rho_{\mathfrak{e}} \tilde{\mathfrak{e}}_{t-1} + \varepsilon_{\mathfrak{e},t} \tag{65}$$

where $\tilde{\mathfrak{e}}_t = \frac{\mathfrak{e}_t}{A_t}$ is the stochastically detrended level of per capita EQC payments, $\rho_{\mathfrak{e}}$ determines the persistence of the spending process and $\varepsilon_{\mathfrak{e},t}$ are shocks added when EQC payments are made. EQC payments will be spent on new investment goods to replace the damaged and destroyed capital stock. The EQC payments process could be more formally linked to private investment, appearing as a proportionate investment demand shock process on new investment goods in the household's budget constraint. This would ensure that EQC payments match the increases in private investment following an earthquake exactly. While I do not do that here, I do make sure that EQC payments are mirrored with increases in private investment of the same size when adding judgements to the model. The medium term debt to GDP target is determined by the following process,

$$\log b_t^\star = \varrho \left(\rho_{b,1} \log b_{t-1}^\star + (1-\rho_{b,1}) \log b_T^\star + \rho_{b,2} \log \left(\frac{b_{t-1}^\star}{b_{t-2}^\star} \right) \right) + (1-\varrho) \log b_{exog,t}^\star, \tag{66}$$

where ϱ takes the value 1 when the target follows an AR 2 process, and 0 when the debt to GDP target is exogenous. A time-varying debt to GDP target is necessary for producing scenarios where the debt to GDP target is changed, like the fiscal consolidation scenarios. I list a number of useful reporting definitions including; per capita total superannuation spending,

$$s_t = w_t d \Psi_t, \tag{67}$$

per capita total tax revenue

$$\mathfrak{t}_t = \tau_t^c c_t + \tau_t^l w_t \ell_t + \tau_t^k \left(R_t \upsilon_t - \omega \delta \right) \left(1 - \varphi \Delta_t \right) \bar{k}_{t-1} \left(1 + n_t \right)^{-1} + \tau^s w_t d \Psi_t, \tag{68}$$

note that z_t is not included in this expression, because $-z_t$ is actually equal to non-superannuation and non-EQC transfers payments. Per capita total expenditure,

$$e_t = g_t - z_t + s_t + \mathfrak{e}_t, \tag{69}$$

where z_t enters negatively in the total expenditure definition for the same reason. The per capita primary surplus,

$$pb_t = t_t - g_t, \tag{70}$$

the per capita operating balance,

$$ob_t = t_t - g_t - r_{t-1} b_{t-1} \left(1 + n_t \right)^{-1} = pb_t - r_{t-1} b_{t-1} \left(1 + n_t \right)^{-1} = b_t - b_{t-1} \left(1 + n_t \right)^{-1}, \tag{71}$$

per capita debt servicing costs,

$$rb_t = r_{t-1} b_{t-1} \left(1 + n_t \right)^{-1}, \tag{72}$$

per capita total tax revenue from capital income,

$$t_t^k = \tau_t^k \left(R_t \upsilon_t - \omega \delta \right) \left(1 - \varphi \Delta_t \right) \bar{k}_{t-1} \left(1 + n_t \right)^{-1}, \tag{73}$$

per capita total tax revenue from consumption expenditure,

$$t_t^c = \tau_t^c c_t, \tag{74}$$

per capita total tax revenue from labour income,

$$t_t^l = \tau_t^l w_t h_t l_t. \tag{75}$$

A.5 Goods Market Clearing and Foreign Block

Intermediate goods can either be consumed domestically, or exported,

$$y_t = y_{H,t} + y^*_{H,t}, \tag{76}$$

where $y^*_{H,t}$ is intermediate export goods in per capita terms. The foreign final goods producer is assumed to produce final goods (a^*_t) according to the technology,

$$a^*_t = \left[\alpha^{*\frac{1}{\varepsilon}} y^{*1-\frac{1}{\varepsilon}}_{F,t} + (1-\alpha^*)^{\frac{1}{\varepsilon}} \left(y^*_{H,t} \left(1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 \right) \right)^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{77}$$

where $y^*_{F,t}$ is the foreign intermediate good used in production of the foreign final good, α^* is the home bias parameter in the foreign economy, and ϕ_X determines the weight on the quadratic adjustment cost, with the foreign final goods producer being subject to a quadratic adjustment cost on imports (exports from New Zealand). Assuming the foreign final goods producer is subject to perfect competition, I obtain the foreign final goods producer's demand for domestically produced intermediate goods by setting up the Lagrangian,

$$\begin{aligned} \mathscr{L}_t &= p^*_{F,t} y^*_{F,t} + p^*_{H,t} y^*_{H,t} - \dots \\ &\dots - p^*_t \left[\left[\alpha^{*\frac{1}{\varepsilon}} y^{*1-\frac{1}{\varepsilon}}_{F,t} + (1-\alpha^*)^{\frac{1}{\varepsilon}} \left(y^*_{H,t} \left(1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 \right) \right)^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} - a^*_t \right], \end{aligned} \tag{78}$$

and differentiaing with respect to imports in the foreign economy,

$$\begin{aligned} \frac{\partial \mathscr{L}_t}{\partial y^*_{H,t}} &= p^*_{H,t} - p^*_t a^{*\frac{1}{\varepsilon}}_t (1-\alpha^*)^{\frac{1}{\varepsilon}} \left(y^*_{H,t} \left(1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 \right) \right)^{-\frac{1}{\varepsilon}} \times \dots \\ &\dots \times \left[1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 - \phi_X \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right) \frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) \right] = 0. \end{aligned} \tag{79}$$

After some rearranging I obtain the demand function for exports,

$$\begin{aligned} \frac{p_t}{q_t} &= (1-\alpha^*)^{\frac{1}{\varepsilon}} \left(\frac{y^*_t}{y^*_{H,t}} \right)^{\frac{1}{\varepsilon}} \left(1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 \right)^{-\frac{1}{\varepsilon}} \times \dots \\ &\dots \times \left[1 - \frac{\phi_X}{2} \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right)^2 - \phi_X \left(\frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) - \mu \right) \frac{y^*_{H,t}}{\bar{y}^*_{H,t-1}} (1+n_t) \right]. \end{aligned} \tag{80}$$

Substituting (43) into (76) gives,

$$y_t = \alpha p_t^{-\varepsilon} (c_t + i_t + g_t) + y^*_{H,t}, \tag{81}$$

where I assume that the final good is either consumed or invested by both the private and the public sectors. Foreign bonds are denominated in domestic currency. As a consequence, the usual uncovered interest parity condition does not drop out of the representative household's first order conditions. If the foreign block were fully specified in this model, the uncovered interest rate condition could be obtained from their first order conditions. To remedy this, I impose a real (uncovered) interest parity condition on the model,

$$1 + r_t = E_t \left\{ \frac{q_{t+1}}{q_t} (1 + r^*_t) \Upsilon_t \mathscr{A}_{q,t} \right\}, \tag{82}$$

where r_t^* is the foreign real interest rate, $\mathcal{A}_{q,t}$ is an autoregressive shock process and Υ_t is a debt elastic interest rate premium used to close the model (see Schmitt-Grohe and Uribe, 2003). The debt elastic interest rate premium is specified as follows,

$$\Upsilon_t = \Upsilon \exp\left(-\phi_F \left(\frac{f_t}{4 \times p_t y_t} - f_t^\star\right)\right), \tag{83}$$

where I allow for a time-varying medium run net foreign asset to GDP target,

$$\log f_t^\star = \rho_f \log f_{t-1}^\star + (1 - \rho_f)\, f_T^\star \tag{84}$$

with $\Upsilon = \frac{1+r}{1+r^*}$, which allows for a permanent wedge between home and foreign real interest rates,[76] and $\tilde{f}_t = \frac{f_t}{A_t}$. Substituting equations (53), (55),

$$p_t y_t = R_t k_t + w_t h_t l_t, \quad \text{and} \quad \xi_t = \left(\gamma_{\upsilon,1}(\upsilon_t - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1 - \varphi \Delta_t)\, \bar{k}_{t-1}(1 + n_t)^{-1},$$

into (7) gives the law of motion for net foreign debt,

$$f_t = (1 + r_{t-1})\, f_{t-1}(1 + n_t)^{-1} + p_t y_t - c_t - i_t - g_t. \tag{85}$$

Net exports, deflated by the final goods deflator, are defined as follows,

$$nx_t = \frac{p_{H,t}}{p_{c,t}} y_{H,t}^* - \frac{\mathfrak{s}_t p_t^*}{p_{c,t}} y_{F,t} = p_t y_{H,t}^* - q_t y_{F,t}. \tag{86}$$

The foreign real interest rate, r^*, and the per capita level of foreign real GDP, y^*, are set exogenously.

76 This is important for matching the model with the data. Interest rates in New Zealand have been higher than trading partner countries' on average which is normally explained by the existence of a country risk premium. Alternative assumptions for closing small open economy models do not necessarily allow for this discrepancy.

B. Stochastically Detrended Model

I stochastically detrend the model so that it can be solved relative to a steady state.[77] This means dividing all per capita variables that grow by the level of labour-augmenting technology. The 47 equations: (2), (3), (9), (18) - (23), (26) - (28), (30), (36), (37), (44), (47), (49), (51), (53), (55), (57) - (75) - (86), along with shock processes for $\mathscr{A}_{c,t}$, $\mathscr{A}_{i,t}$, $\mathscr{A}_{q,t}$, $\mathscr{A}_{g_h,t}$, $\mathscr{A}_{g_i,t}$, $\mathscr{A}_{g_g,t}$, $\mathscr{A}_{z,t}$, $\mathscr{A}_{x,t}$, $\mathscr{A}_{m,t}$, $\mathscr{A}_{b,t}$ and the definition, $k_t = (1-\varphi\Delta_t)\,\upsilon_t \bar{k}_{t-1}(1+n_t)^{-1}$, describe the model's dynamic equilibrium for the 58 variables: $\tilde{c}_t = \frac{c_t}{A_t}$, l_t, τ^c_t, τ^k_t, τ^l_t, $\tilde{i}_t = \frac{i_t}{A_t}$, $\tilde{\bar{k}}_t = \frac{\bar{k}_t}{A_t}$, $\tilde{k}_t = \frac{K_t}{A_t}$, υ_t, $\tilde{b}_t = \frac{b_t}{A_t}$, $\tilde{f}_t = \frac{f_t}{A_t}$, r_t, R_t, $\tilde{w}_t = \frac{w_t}{A_t}$, $\tilde{t}_t = \frac{t_t}{A_t}$, $\tilde{g}_t = \frac{g_t}{A_t}$, $\tilde{g}_{h,t} = \frac{g_{h,t}}{A_t}$, $\tilde{g}_{i,t} = \frac{g_{i,t}}{A_t}$, $\tilde{g}_{g,t} = \frac{g_{g,t}}{A_t}$, $\tilde{\lambda}_t = \lambda_t A_t$, $\tilde{z}_t = \frac{z_t}{A_t}$, $\tilde{\mathfrak{e}}_t = \frac{\mathfrak{e}_t}{A_t}$ $b^\star_t$, $f^\star_t$, $y^*_{H,t}$, $\tilde{y}_t = \frac{y_t}{A_t}$, γ_t, q_t, p_t, Γ_t, $\tilde{k}_{G,t} = \frac{K_{G,t}}{A_t}$, $\mathscr{A}_t$, $\mathscr{A}_{\gamma,t}$, $\mathscr{A}_{c,t}$, $\mathscr{A}_{i,t}$, $\mathscr{A}_{q,t}$, $\mathscr{A}_{g_h,t}$, $\mathscr{A}_{g_i,t}$, $\mathscr{A}_{g_g,t}$, $\mathscr{A}_{z,t}$, $\mathscr{A}_{x,t}$, $\mathscr{A}_{m,t}$, $\mathscr{A}_{b,t}$, κ_t, Φ_t, Υ_t, $\tilde{nx}_t = \frac{nx_t}{A_t}$, $\tilde{y}_{F,t} = \frac{y_{F,t}}{A_t}$, $\tilde{s}_t = \frac{s_t}{A_t}$, $\tilde{c}^\star_t = \frac{c^\star_t}{A_t}$, $\tilde{\mathfrak{t}}_t = \frac{\mathfrak{t}_t}{A_t}$, $\tilde{pb}_t = \frac{pb_t}{A_t}$, $\tilde{ob}_t = \frac{ob_t}{A_t}$, $\tilde{rb}_t = \frac{rb_t}{A_t}$, $\tilde{t}^k_t = \frac{t^k_t}{A_t}$, $\tilde{t}^c_t = \frac{t^c_t}{A_t}$, $\tilde{t}^l_t = \frac{t^l}{A_t}$, $\tilde{e}_t = \frac{e_t}{A_t}$,

$$\tilde{\lambda}_t \exp(\mathscr{A}_{b,t}) = E_t\left\{\beta\tilde{\lambda}_{t+1}(1+\gamma_{t+1})^{-1}(1+r_t)\right\}, \tag{87}$$

$$\tilde{\lambda}_t = \frac{\mathscr{A}_{c,t}\left(\tilde{c}^\star_t - \chi\tilde{c}^\star_{t-1}\right)^{-\sigma}}{1+\tau^c_t}, \tag{88}$$

$$\tilde{c}^\star_t = \tilde{c}_t + \alpha_{g_g}\tilde{g}_{g,t} + \alpha_{g_i}\tilde{g}_{i,t} + \alpha_{g_h}\tilde{g}_{h,t}, \tag{89}$$

$$\tilde{\lambda}_t\tilde{w}_t = (1-\chi)^{-\sigma}\varkappa_t\kappa_t\left(\frac{l^\eta_t}{1-\tau^l_t}\right), \tag{90}$$

$$\Phi_t = E_t\left\{\begin{array}{c}\beta\dfrac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+\gamma_{t+1})^{-1}(1-\varphi\Delta_{t+1})\times\ldots\\ \ldots\times\left[\begin{array}{c}\left(1-\tau^k_{t+1}\right)R_{t+1}\upsilon_{t+1} - \gamma_{\upsilon,1}(\upsilon_{t+1}-1) - \dfrac{\gamma_{\upsilon,2}}{2}(\upsilon_{t+1}-1)^2 + \ldots\\ \ldots + \omega\tau^k_{t+1}\delta + \Phi_{t+1}(1-\delta)\end{array}\right]\end{array}\right\}, \tag{91}$$

$$\begin{aligned}1 = \Phi_t&\left[\begin{array}{c}1 - \dfrac{\phi}{2}\left(\dfrac{\mathscr{A}_{i,t}\tilde{i}_t}{\tilde{i}_{t-1}}(1+\gamma_t)(1+n_t) - \mu\right)^2 - \ldots\\ \ldots - \phi\left(\dfrac{\mathscr{A}_{i,t}\tilde{i}_t}{\tilde{i}_{t-1}}(1+\gamma_t)(1+n_t) - \mu\right)\dfrac{\mathscr{A}_{i,t}\tilde{i}_t}{\tilde{i}_{t-1}}(1+\gamma_t)(1+n_t)\end{array}\right] + \ldots\\ \ldots + E_t&\left\{\begin{array}{c}\beta\dfrac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+\gamma_{t+1})^{-1}\Phi_{t+1}\phi\left(\dfrac{\mathscr{A}_{i,t+1}\tilde{i}_{t+1}}{\tilde{i}_t}(1+\gamma_{t+1})(1+n_{t+1}) - \mu\right)\times\cdots\\ \cdots\times\left(\dfrac{\mathscr{A}_{i,t+1}\tilde{i}_{t+1}}{\tilde{i}_t}(1+\gamma_{t+1})(1+n_{t+1})\right)^2\end{array}\right\},\end{aligned} \tag{92}$$

$$\left(1-\tau^k_t\right)R_t = \gamma_{\upsilon,1} + \gamma_{\upsilon,2}(\upsilon_t - 1), \tag{93}$$

[78]

$$\tilde{k}_t = (1-\varphi\Delta_t)\,\upsilon_t\tilde{\bar{k}}_{t-1}(1+n_t)^{-1}(1+\gamma_t)^{-1}, \tag{94}$$

$$\theta p_t\frac{\tilde{y}_t}{\tilde{k}_t} = R_t, \tag{95}$$

77 In principle I could solve the model around a balanced growth path without detrending the model. However, because the balanced growth path is growing, the associated residuals, which both the deterministic solution and the extended path algorithm are trying to minimise, also grow in size with the projection horizon. This could lead to numerical issues with the algorithm, which are not present when the stochastically detrended model is used.

78 When modelling transition paths, I replace this equation with $\upsilon_t = 1$, switching off variable capacity utilisation. This is equivalent to using a large value for $\gamma_{\upsilon,2}$.

$$(1-\theta)\, p_t \frac{\tilde{y}_t}{h_t l_t} = \tilde{w}_t, \tag{96}$$

$$\tilde{b}_t = \tilde{g}_t - \tilde{t}_t + (1+r_{t-1})\, \tilde{b}_{t-1} (1+\gamma_t)^{-1} (1+n_t)^{-1}, \tag{97}$$

$$\tilde{k}_{G,t} = (1-\varphi_G \Delta_t)(1-\delta_G)\, \tilde{k}_{G,t-1} (1+n_t)^{-1} (1+\gamma_t)^{-1} + \tilde{g}_{i,t}, \tag{98}$$

$$\tilde{y}_t = \mathcal{A}_t \tilde{k}_t^{\theta} (h_t l_t)^{1-\theta} \left((1-\varphi_G \Delta_t)\, \tilde{k}_{G,t-1} (1+n_t)^{-1} (1+\gamma_t)^{-1} \right)^{\theta_G}, \tag{99}$$

$$\tilde{\tilde{k}}_t = \left[1 - \frac{\phi}{2} \left(\frac{\mathcal{A}_{i,t} \tilde{i}_t}{\tilde{i}_{t-1}} (1+\gamma_t)(1+n_t) - \mu \right)^2 \right] \tilde{i}_t + (1-\varphi\Delta_t)(1-\delta)\, \tilde{\tilde{k}}_{t-1} (1+\gamma_t)^{-1} (1+n_t)^{-1}, \tag{100}$$

$$\tilde{y}_t = \alpha p_t^{-\varepsilon} \left(\tilde{c}_t + \tilde{i}_t + \tilde{g}_t \right) + \tilde{y}^*_{H,t}, \tag{101}$$

$$\begin{aligned} \frac{p_t}{q_t} = (1-\alpha^*)^{\frac{1}{\varepsilon}} \left(\frac{\tilde{y}^*_t}{\tilde{y}^*_{H,t}} \right)^{\frac{1}{\varepsilon}} \left(1 - \frac{\phi_X}{2} \left(\frac{\tilde{y}^*_{H,t}}{\tilde{y}^*_{H,t-1}} (1+\gamma_t)(1+n_t) - \mu \right)^2 \right)^{-\frac{1}{\varepsilon}} \times \ldots \\ \ldots \times \left[\begin{array}{c} 1 - \frac{\phi_X}{2} \left(\frac{\tilde{y}^*_{H,t}}{\tilde{y}^*_{H,t-1}} (1+\gamma_t)(1+n_t) - \mu \right)^2 - \ldots \\ \ldots - \phi_X \left(\frac{\tilde{y}^*_{H,t}}{\tilde{y}^*_{H,t-1}} (1+\gamma_t)(1+n_t) - \mu \right) \frac{\tilde{y}^*_{H,t}}{\tilde{y}^*_{H,t-1}} (1+\gamma_t)(1+n_t) \end{array} \right], \end{aligned} \tag{102}$$

$$p_t = \left[\alpha + (1-\alpha)\, \Gamma_t^{1-\varepsilon} \right]^{\frac{1}{\varepsilon-1}}, \tag{103}$$

$$q_t = \Gamma_t p_t, \tag{104}$$

$$1 + r_t = E_t \left\{ \frac{q_{t+1}}{q_t} (1+r^*_t)\, \Upsilon_t \mathcal{A}_{q,t} \right\}, \tag{105}$$

$$\Upsilon_t = \Upsilon \exp \left(-\phi_F \left(\frac{f_t}{4 \times p_t y_t} - f^\star_t \right) \right), \tag{106}$$

$$\tilde{f}_t = (1+r_{t-1})\, \tilde{f}_{t-1} (1+\gamma_t)^{-1} (1+n_t)^{-1} + p_t \tilde{y}_t - \tilde{c}_t - \tilde{i}_t - \tilde{g}_t, \tag{107}$$

$$\begin{aligned} \tilde{t}_t = \tau^c_t \tilde{c}_t + \tau^l_t \tilde{w}_t h_t l_t + \tau^k_t (R_t v_t - \omega\delta)(1-\varphi\Delta_t)\, \tilde{\tilde{k}}_{t-1} (1+n_t)^{-1} (1+\gamma_t)^{-1} - \ldots \\ \ldots - (1-\tau^s)\, \tilde{w}_t d\Psi_t + \tilde{z}_t - \tilde{\mathfrak{e}}_t, \end{aligned} \tag{108}$$

$$\tilde{g}_t = \tilde{g}_{h,t} + \tilde{g}_{i,t} + \tilde{g}_{g,t}, \tag{109}$$

$$\tilde{g}_{h,t} = \vartheta_{h,t} p_t \tilde{y}_t \mathcal{A}_{g_h,t}, \tag{110}$$

$$\tilde{g}_{i,t} = \vartheta_i p_t \tilde{y}_t \mathcal{A}_{g_i,t}, \tag{111}$$

$$\tilde{g}_{g,t} = \vartheta_{g,t} p_t \tilde{y}_t \mathcal{A}_{g_g,t}, \tag{112}$$

$$\log \left(\frac{\tau^k_t}{\tau^{k\star}_t} \right) = \psi_{\tau^k} \left(\frac{\tilde{b}_{t-1}}{4 \times p_{t-1} \tilde{y}_{t-1}} - b^\star_t \right), \tag{113}$$

$$\log \left(\frac{\tau^c_t}{\tau^{c\star}_t} \right) = \psi_{\tau^c} \left(\frac{\tilde{b}_{t-1}}{4 \times p_{t-1} \tilde{y}_{t-1}} - b^\star_t \right), \tag{114}$$

$$\log \left(\frac{\tau^l_t}{\tau^{l\star}_t} \right) = \psi_{\tau^l} \left(\frac{\tilde{b}_{t-1}}{4 \times p_{t-1} \tilde{y}_{t-1}} - b^\star_t \right), \tag{115}$$

$$\tilde{z}_t = \vartheta_{z,t} p_t \tilde{y}_t \mathcal{A}_{z,t}, \tag{116}$$

$$\tilde{\mathfrak{e}}_t = \rho_{\mathfrak{e}} \tilde{\mathfrak{e}}_{t-1} + \varepsilon_{\mathfrak{e},t}, \tag{117}$$

$$\log b^\star_t = \varrho \left(\rho_{b,1} \log b^\star_{t-1} + (1-\rho_{b,1}) \log b^\star_T + \rho_{b,2} \log \left(\frac{b^\star_{t-1}}{b^\star_{t-2}} \right) \right) + (1-\varrho) \log b^\star_{exog,t}, \tag{118}$$

$$\log f_t^\star = \rho_f \log f_{t-1}^\star + (1-\rho_f) \log f_T^\star, \tag{119}$$

$$1+\gamma_t = (1+\gamma) \exp\left(\mathscr{A}_{\gamma,t} - \mathscr{A}_{\gamma,t-1}\right), \tag{120}$$

$$\mathscr{A}_{\gamma,t} = \rho_{\mathscr{A}_\gamma} \mathscr{A}_{\gamma,t-1} + \varepsilon_{\gamma,t}, \tag{121}$$

$$\log \mathscr{A}_t = \rho_{\mathscr{A}} \log \mathscr{A}_{t-1} + \varepsilon_{\mathscr{A},t}, \tag{122}$$

$$\log \mathscr{A}_{c,t} = \rho_{\mathscr{A}_c} \log \mathscr{A}_{c,t-1} + \varepsilon_{\mathscr{A}_C,t}, \tag{123}$$

$$\log \mathscr{A}_{i,t} = \rho_{\mathscr{A}_i} \log \mathscr{A}_{i,t-1} + \varepsilon_{\mathscr{A}_i,t}, \tag{124}$$

$$\log \mathscr{A}_{q,t} = \rho_{\mathscr{A}_q} \log \mathscr{A}_{q,t-1} + \varepsilon_{\mathscr{A}_q,t}, \tag{125}$$

$$\log \mathscr{A}_{g_h,t} = \rho_{\mathscr{A}_{g_h}} \log \mathscr{A}_{g_h,t-1} + \varepsilon_{\mathscr{A}_{g_h},t}, \tag{126}$$

$$\log \mathscr{A}_{g_i,t} = \rho_{\mathscr{A}_{g_i}} \log \mathscr{A}_{g_i,t-1} + \varepsilon_{\mathscr{A}_{g_i},t}, \tag{127}$$

$$\log \mathscr{A}_{g_g,t} = \rho_{\mathscr{A}_{g_g}} \log \mathscr{A}_{g_g,t-1} + \varepsilon_{\mathscr{A}_{g_g},t}, \tag{128}$$

$$\log \mathscr{A}_{z,t} = \rho_{\mathscr{A}_z} \log \mathscr{A}_{z,t-1} + \varepsilon_{\mathscr{A}_z,t}, \tag{129}$$

$$\log \mathscr{A}_{x,t} = \rho_{\mathscr{A}_x} \log \mathscr{A}_{x,t-1} + \varepsilon_{\mathscr{A}_x,t}, \tag{130}$$

$$\log \mathscr{A}_{m,t} = \rho_{\mathscr{A}_m} \log \mathscr{A}_{m,t-1} + \varepsilon_{\mathscr{A}_m,t}, \tag{131}$$

$$\mathscr{A}_{b,t} = \rho_{\mathscr{A}_b} \mathscr{A}_{b,t-1} + \varepsilon_{\mathscr{A}_b,t}, \tag{132}$$

$$\log \kappa_t = \rho_\kappa \log \kappa_{t-1} + (1-\rho_\kappa) \log \kappa + \varepsilon_{\kappa,t}, \tag{133}$$

$$\tilde{nx}_t = p_t \tilde{y}_{H,t}^* - q_t \tilde{y}_{F,t} \tag{134}$$

$$\tilde{y}_{F,t} = (1-\alpha)\, q_t^{-\varepsilon} \left(\tilde{c}_t + \tilde{i}_t + \tilde{g}_t\right), \tag{135}$$

$$\tilde{s}_t = \tilde{w}_t d\Psi_t, \tag{136}$$

$$\tilde{\mathfrak{t}}_t = \tau_t^c \tilde{c}_t + \tau_t^l \tilde{w}_t h_t l_t + \tau_t^k \left(R_t \upsilon_t - \omega\delta\right)\left(1-\varphi\Delta_t\right) \tilde{\tilde{k}}_{t-1} \left(1+n_t\right)^{-1} \left(1+\gamma_t\right)^{-1} + \tau^s \tilde{w}_t d\Psi_t, \tag{137}$$

$$\tilde{pb}_t = \tilde{t}_t - \tilde{g}_t, \tag{138}$$

$$\tilde{ob}_t = \tilde{t}_t - \tilde{g}_t - r_{t-1} \tilde{b}_{t-1} \left(1+\gamma_t\right)^{-1} \left(1+n_t\right)^{-1}, \tag{139}$$

$$\tilde{rb}_t = r_{t-1} \tilde{b}_{t-1} \left(1+\gamma_t\right)^{-1} \left(1+n_t\right)^{-1}, \tag{140}$$

$$\tilde{t}_t^k = \tau_t^k \left(R_t \upsilon_t - \omega\delta\right)\left(1-\varphi\Delta_t\right) \tilde{\tilde{k}}_{t-1} \left(1+n_t\right)^{-1} \left(1+\gamma_t\right)^{-1}, \tag{141}$$

$$\tilde{t}_t^c = \tau_t^c \tilde{c}_t, \tag{142}$$

$$\tilde{t}_t^l = \tau_t^l \tilde{w}_t \ell_t. \tag{143}$$

$$\tilde{e}_t = \tilde{g}_t + \tilde{s}_t - \tilde{z}_t + \tilde{\mathfrak{e}}_t. \tag{144}$$

The variables; h_t, $\varkappa_t$, $\vartheta_{h,t}$, $\vartheta_{g,t}$, $\tau_t^{l\star}$, $\tau_t^{k\star}$, $\tau_t^{c\star}$, $\vartheta_{z,t}$, n_t, Ψ_t, r_t^*, $\tilde{y}_t^*$, $b_{exog,t}^\star$ are set exogenously. In the first period of the transition, they will be a surprise for agents in the model. In all subsequent periods, they will be perfectly anticipated, in the baseline scenario.

C. Finding the Initial Steady State

Both the deterministic perfect foresight solution and the extended path algorithm require an initial condition and a terminal condition. In this appendix I describe how I find the model's initial steady state, which I use for my initial condition in the projections. The initial steady state is chosen to be close to current economic conditions. I treat the following 42 variables as endogenous and solve for their steady state values: $\tilde{c}_0$, $\tilde{i}_0$, $\tilde{\tilde{k}}_0$, $\tilde{k}_0$, $\tilde{b}_0$, $\tilde{f}_0$, r_0, R_0, $\tilde{w}_0$, $\tilde{t}_0$, $\tilde{g}_0$, $\tilde{g}_{h,0}$, $\tilde{g}_{i,0}$, $\tilde{g}_{g,0}$, $\tilde{\lambda}_0$, $\tilde{z}_0$, $b_0^\star$, $y_{H,0}^*$, $\tilde{y}_0$, q_0, p_0, $\tilde{k}_{G,0}$, κ_0, Φ_0, Υ_0, $\tilde{nx}_0$, $\tilde{y}_{F,0}$, $\tilde{s}_0$, $\tilde{c}_0^\star$, $\tilde{\mathfrak{t}}_0$, $\tilde{pb}_0$, $\tilde{ob}_0$, $\tilde{rb}_0$, $\tilde{t}_0^k$, $\tilde{t}_0^c$, $\tilde{t}_0^l$, $\tilde{e}_0$, $\tilde{y}_0^*$, $\tau_0^{l\star}$, $\tau_0^{k\star}$, $\tau_0^{c\star}$, $\vartheta_{z,0}$,

Most of the shock processes; $\mathscr{A}_0$, $\mathscr{A}_{\gamma,0}$, $\mathscr{A}_{c,0}$, $\mathscr{A}_{i,0}$, $\mathscr{A}_{q,0}$, $\mathscr{A}_{g_h,0}$, $\mathscr{A}_{g_i,0}$, $\mathscr{A}_{g_g,0}$, $\mathscr{A}_{x,0}$, $\mathscr{A}_{m,0}$, $\mathscr{A}_{b,0}$, are normalised, taking the value of 1 or 0 in the steady state: $\mathscr{A}_0 = 1$, $\mathscr{A}_{j,0} = 1$ for $j = c$, i, q, g_h, g_i, g_g, z, m, and $\mathscr{A}_{j,0} = 0$ for $j = \gamma$, b. I treat the following variables as either exogenous or I exogenise them in order to solve the initial steady state: υ_0, $\tilde{\mathfrak{e}}_0$, $f_0^\star$, τ_0^c, τ_0^k, τ_0^l, Γ_0, γ_0, ϑ_i, $\vartheta_{h,0}$, $\vartheta_{g,0}$, Ψ_0, $b_{exog,0}^\star$ h_0, $\varkappa_0$, n_0, r_0^*, l_0. I exogenise the initial value of labour, l_0, and I endogenise the initial value for the weight on the disutility of labour, κ_0. This allows me to choose a sensible value of l_0 and find the value of κ_0 that is consistent with this choice in the initial steady state. I also exogenise initial values of the net debt to GDP target, $b_{exog}^\star$, the average tax rates, τ_0^c, τ_0^k and τ_0^l, and the expenditure ratios, ϑ_i, $\vartheta_{h,0}$ and $\vartheta_{g,0}$. I determine the initial value of non-superannuation transfers to GDP, $\vartheta_{z,0}$, as the residual from the government's budget constraint.

Steady state capacity utilisation, υ_0, is normalised to 1 so that capacity utilisation and the associated adjustment costs drop out of the steady state model. In steady state there are no EQC payments, so that $\tilde{\mathfrak{e}}_0 = 0$. I normalise the terms of trade in the initial steady state, Γ_0, setting it equal to 1. Equation (120) implies that the initial value for total factor productivity growth should be $\gamma_0 = \gamma$. The initial condition for the exogenous human capital, h_0, is set to 1. The age-related disutility of working, $\varkappa_0$, is normalised to 1 in the initial steady state. Population growth, n_0, is chosen to match the initial value of the exogenous population track described in Section 4. The initial value for the long-run net foreign asset to GDP ratio, $f_0^\star$, is chosen to loosely match the average over history.

The initial condition for the net debt to GDP target, $b_0^\star = b_{exog,0}^\star$, is set to 48 percent, the peak of net debt to GDP over the forecast period, as described in Section 4. The initial value of the old-age ratio (the ratio of the population over the age of 65 to the population over the age of 15), Ψ_0, is set to match 2021 values from the data, while the process for setting the initial values of the spending ratios, ϑ_i, $\vartheta_{h,0}$ and $\vartheta_{g,0}$ is described in Section 4. The calibration of the initial average tax rates, τ_0^c, τ_0^k and τ_0^l is discussed in Appendix F.

The initial public capital to GDP ratio can be solved using equation (98) will all time subscripts set to period zero,

$$\tilde{k}_{G,0} = (1 - \delta_G)\,\tilde{k}_{G,0}\,(1 + n_0)^{-1}\,(1 + \gamma_0)^{-1} + \tilde{g}_{i,0}, \tag{145}$$

$$\left(\frac{\tilde{k}_G}{p\tilde{y}}\right)_0 = \vartheta_{g_i}\left(1 - \frac{1 - \delta_G}{(1 + n_0)\,(1 + \gamma_0)}\right)^{-1}, \tag{146}$$

where $\left(\frac{\tilde{k}_G}{p\tilde{y}}\right)_0$ is an expression for the initial public capital to GDP ratio. With the terms of trade exogenised and normalised to 1, I can obtain a value for the steady state relative price level,

$$p_0 = \left[\alpha + (1 - \alpha)\,\Gamma_0^{1-\varepsilon}\right]^{\frac{1}{\varepsilon - 1}} = 1, \tag{147}$$

and the real exchange rate,

$$q_0 = \Gamma_0 p_0 = 1. \tag{148}$$

I calibrate the discount factor to match the initial steady state productivity growth and real interest rate assumptions. The initial value for the real interest rate can be obtained from the steady state Euler equation (87),

$$r_0 = \frac{\beta}{1+\gamma_0} - 1. \tag{149}$$

The steady state value of the risk premium is chosen to account for discrepancies between the intial values of home and foreign real interest rates, as implied by the real UIP condition (105),

$$\Upsilon_0 = \left(\frac{1+r_0}{1+r_0^{\star}}\right). \tag{150}$$

I use the asset pricing equation/Tobin's Q relationship (91), to find the initial steady state capital output ratio. I start by setting the time subscripts to period zero,

$$\Phi_0 = \beta\frac{\tilde{\lambda}_0}{\tilde{\lambda}_0}\left(1+\gamma_0\right)^{-1}\left[\begin{array}{c}\left(1-\tau_0^k\right)R_0\upsilon_0 - \gamma_{\upsilon,1}\left(\upsilon_0-1\right) - \frac{\gamma_{\upsilon,2}}{2}\left(\upsilon_0-1\right)^2 + \ldots \\ \ldots + \omega\tau_0^k\delta + \Phi_0\left(1-\delta\right)\end{array}\right]. \tag{151}$$

Equation (92) implies that $\Phi_0 = 1$ in steady state. Substituting this and $R_0 = \theta\frac{p_0\tilde{y}_0}{\tilde{\tilde{k}}_0}\left(1+\gamma_0\right)\left(1+n_0\right) = \theta\left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0^{-1}\left(1+\gamma_0\right)\left(1+n_0\right)$ into (151) gives,

$$1 = \beta\left(1+\gamma_0\right)^{-1}\left[\left(1-\tau_0^k\right)\theta\left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0^{-1}\left(1+\gamma_0\right)\left(1+n_0\right) + \omega\tau_0^k\delta + 1 - \delta\right], \tag{152}$$

where υ_0 drops out because $\upsilon_0 = 1$ in steady state. Rearranging for $\left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0$,

$$\left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0 = \frac{\theta\beta\left(1-\tau_0^k\right)\left(1+\gamma_0\right)\left(1+n_0\right)}{1+\gamma_0-\beta\left(1-\delta\right)-\beta\omega\tau_0^k\delta}, \tag{153}$$

where it follows,

$$\left(\frac{\tilde{k}}{p\tilde{y}}\right)_0 = \left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0\left(1+\gamma_0\right)^{-1}\left(1+n_0\right)^{-1} = \frac{\theta\beta\left(1-\tau_0^k\right)}{1+\gamma_0-\beta\left(1-\delta\right)-\beta\omega\tau_0^k\delta}. \tag{154}$$

Using the intermediate sector's production function, steady state output can be written as a function of the private capital output ratio, the government capital output ratio and the exogenous initial steady state value for labour, l_0. Starting with the production function (99) and setting the time subscripts to period 0,

$$\tilde{y}_0 = \mathscr{A}_0\tilde{k}_0^{\theta}\left(h_0l_0\right)^{1-\theta}\left(\tilde{k}_{G,0}\left(1+n_0\right)^{-1}\left(1+\gamma_0\right)^{-1}\right)^{\theta_G}. \tag{155}$$

Substituting in the private capital and government capital output ratios, (153) and (146), gives,

$$\tilde{y}_0 = \left(p_0\tilde{y}_0\right)^{\theta+\theta_G}\mathscr{A}_0\left(\frac{\tilde{k}}{p\tilde{y}}\right)_0^{\theta}\left(h_0l_0\right)^{1-\theta}\left(\left(\frac{\tilde{k}_G}{p\tilde{y}}\right)_0\left(1+n_0\right)^{-1}\left(1+\gamma_0\right)^{-1}\right)^{\theta_G}. \tag{156}$$

Rearranging for output,

$$\tilde{y}_0 = \left[p_0^{\theta+\theta_G}\mathscr{A}_0\left(\frac{\tilde{k}}{p\tilde{y}}\right)_0^{\theta}\left(h_0l_0\right)^{1-\theta}\left(\left(\frac{\tilde{k}_G}{p\tilde{y}}\right)_0\left(1+n_0\right)^{-1}\left(1+\gamma_0\right)^{-1}\right)^{\theta_G}\right]^{\frac{1}{1-\theta-\theta_G}}. \tag{157}$$

I now have an expression for the steady state level of stochastically detrended output that is consistent with my initial condition for labour, l_0. Using the initial values for p_0 and $\tilde{y}_0$ in combination with (146) I obtain an initial value for the government capital stock,

$$\tilde{k}_{G,0} = \left(\frac{\tilde{k}_G}{p\tilde{y}}\right)_0 p_0\tilde{y}_0. \tag{158}$$

Using the initial values for p_0 and $\tilde{y}_0$ in combination with (153) I obtain an initial value for the private capital stock,

$$\tilde{\tilde{k}}_0 = \left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0 p_0\tilde{y}_0. \tag{159}$$

The initial values for p_0 and $\tilde{y}_0$ can be combined with the initial values of the net debt to GDP ratio, the net foreign asset to GDP ratio, and the government expenditure ratios to get

$$\tilde{b}_0 = 4 \times b_0^\star p_0\tilde{y}_0, \tag{160}$$

$$\tilde{f}_0 = 4 \times f_0^\star p_0\tilde{y}_0, \tag{161}$$

$$\tilde{g}_{i,0} = \vartheta_{g_i} p_0\tilde{y}_0 \mathcal{A}_{g_i,0}, \tag{162}$$

$$\tilde{g}_{h,0} = \vartheta_{g_h,0} p_0\tilde{y}_0 \mathcal{A}_{g_h,0}, \tag{163}$$

$$\tilde{g}_{g,0} = \vartheta_{g_g,0} p_0\tilde{y}_0 \mathcal{A}_{g_g,0}, \tag{164}$$

where $\tilde{b}_0^\star$, $\tilde{f}_0^\star$, ϑ_{g_i}, $\vartheta_{g_h,0}$ and $\vartheta_{g_g,0}$ are exogenously determined. With $\tilde{g}_{i,0}$, $\tilde{g}_{h,0}$ and $\tilde{g}_{g,0}$ known, I can use equation (109) to obtain,

$$\tilde{g}_0 = \tilde{g}_{i,0} + \tilde{g}_{g,0} + \tilde{g}_{h,0}. \tag{165}$$

With the initial value of the private capital stock known, I can set the time subscripts in the capital accumulation equation (100) to zero and solve for the initial value of private investment,

$$\tilde{\tilde{k}}_0 = \tilde{i}_0 + (1-\delta)\,\tilde{\tilde{k}}_0\,(1+\gamma_0)^{-1}\,(1+n_0)^{-1}, \tag{166}$$

$$\tilde{i}_0 = \left[1 - (1-\delta)\,(1+\gamma_0)^{-1}\,(1+n_0)^{-1}\right]\tilde{\tilde{k}}_0. \tag{167}$$

The initial value of consumption can be obtained from the aggregate demand condition (107), by setting the time subscript to period 0 and rearranging for consumption,

$$\tilde{f}_0 = (1+r_0)\,\tilde{f}_0\,(1+\gamma_0)^{-1}\,(1+n_0)^{-1} + p_0\tilde{y}_0 - \tilde{c}_0 - \tilde{i}_0 - \tilde{g}_0, \tag{168}$$

$$\tilde{c}_0 = \tilde{y}_0 - \tilde{i}_0 - \tilde{g}_0 + \left(\frac{1+r_0}{(1+\gamma_0)\,(1+n_0)} - 1\right)\tilde{f}_0. \tag{169}$$

Likewise, the initial values for effective consumption and the marginal utility of consumption can be obtained from equations (89) and (88), respectively,

$$\tilde{c}_0^\star = \tilde{c}_0 + \alpha_{g_g}\tilde{g}_{g,0} + \alpha_{g_i}\tilde{g}_{i,0} + \alpha_{g_h}\tilde{g}_{h,0}, \tag{170}$$

$$\tilde{\lambda}_0 = \frac{\mathcal{A}_{c,0}\,(\tilde{c}_0^\star - \chi\tilde{c}_0^\star)^{-\sigma}}{1+\tau_0^c}. \tag{171}$$

Substituting $\tilde{nx}_0$ for $p_0\tilde{y}_0 - \tilde{c}_0 - \tilde{i}_0 - \tilde{g}_0$ in equation (172) and rearranging for $\tilde{nx}_0$,

$$\tilde{f}_0 = (1+r_0)\,\tilde{f}_0\,(1+\gamma_0)^{-1}\,(1+n_0)^{-1} + \tilde{nx}_0, \tag{172}$$

$$\tilde{nx}_0 = \left(1 - (1+r_0)\,(1+\gamma_0)^{-1}\,(1+n_0)^{-1}\right)\tilde{f}_0. \tag{173}$$

The initial value for imports is obtained from equation (135),

$$\tilde{y}_{F,0} = (1-\alpha)\,q_0^{-\varepsilon}\left(\tilde{c}_0 + \tilde{i}_0 + \tilde{g}_0\right). \tag{174}$$

I can find the level of labour productivity as a function of the capital output ratio, by dividing the production function (155) by $\tilde{y}_t$,

$$1 = p_0\mathcal{A}_0\left(\frac{\tilde{k}}{p\tilde{y}}\right)_0^{\theta}\left(\frac{p\tilde{y}}{l}\right)_0^{\theta-1} h_0^{1-\theta}\left(\tilde{k}_{G,0}\,(1+n_0)^{-1}\,(1+\gamma_0)^{-1}\right)^{\theta_G}, \tag{175}$$

and then rearranging to obtain,

$$\left(\frac{p\tilde{y}}{l}\right)_0 = p_0\mathscr{A}_0^{\frac{1}{1-\theta}}\left(\frac{\tilde{k}}{p\tilde{y}}\right)_0^{\frac{\theta}{1-\theta}} h_0\left(\tilde{k}_{G,0}\left(1+n_0\right)^{-1}\left(1+\gamma_0\right)^{-1}\right)^{\frac{\theta_G}{1-\theta}}. \tag{176}$$

I can use equations (90), (88) and (96) to find κ_0, the weight on the disutility of labour, that is consistent with the initial value for labour, l_0,

$$\tilde{w}_0\left(1-\tau_0^l\right) = \left(1+\tau_0^c\right)\varkappa_0\kappa_0\tilde{c}_t^\sigma l_0^\eta, \tag{177}$$

$$\tilde{w}_0 = (1-\theta)\frac{p_0\tilde{y}_0}{h_0 l_0} = (1-\theta)\left(\frac{p\tilde{y}}{l}\right)_0 h_0^{-1}, \tag{178}$$

$$(1-\theta)\left(\frac{p\tilde{y}}{l}\right)_0 = \left(\frac{1+\tau_0^c}{1-\tau_0^l}\right)\varkappa_0\kappa_0\tilde{c}_t^\sigma l_0^\eta, \tag{179}$$

$$\kappa_0 = \frac{(1-\theta)\left(\frac{p\tilde{y}}{l}\right)_0}{\left(\frac{1+\tau_0^c}{1-\tau_0^l}\right)\varkappa_0\tilde{c}_t^\sigma l_0^\eta}. \tag{180}$$

With the real wage determined by equation (178), I can use the defintion for superannuation payments to obtain,

$$\tilde{s}_0 = \tilde{w}_0 d\Psi_0. \tag{181}$$

Next, I obtain steady state foreign GDP using equations (101) and (102) with the time subscripts set to zero,

$$\tilde{y}_0 = \alpha p_0^{-\varepsilon}\left(\tilde{c}_0+\tilde{i}_0+\tilde{g}_0\right)+\tilde{y}_{H,0}^*, \tag{182}$$

$$\frac{p_0}{q_0} = \left(1-\alpha^*\right)^{\frac{1}{\varepsilon}}\left(\frac{\tilde{y}_0^*}{\tilde{y}_{H,0}^*}\right)^{\frac{1}{\varepsilon}}. \tag{183}$$

Plugging equation (183) into equation (182), I obtain,

$$\tilde{y}_0 = \alpha\left(\tilde{c}_0+\tilde{i}_0+\tilde{g}_0\right)+\left(1-\alpha^*\right)\left(\frac{p_0}{q_0}\right)^{-\varepsilon}\tilde{y}_0^*. \tag{184}$$

I rearrange this to obtain,

$$\tilde{y}_0^* = \frac{\tilde{y}_0-\alpha\left(\tilde{c}_0+\tilde{i}_0+\tilde{g}_0\right)}{\left(1-\alpha^*\right)\left(\frac{p_0}{q_0}\right)^{-\varepsilon}}. \tag{185}$$

Using equation (183), I obtain the initial steady state value for exports,

$$\tilde{y}_{H,0}^* = \left(1-\alpha^*\right)\left(\frac{p_0}{q_0}\right)^{-\varepsilon}\tilde{y}_0^*. \tag{186}$$

Using equations (95) and (153), I can pin down the initial value for the rental/dividend rate on capital,

$$R_0 = \theta\left(\frac{\tilde{\tilde{k}}}{p\tilde{y}}\right)_0^{-1}\left(1+n_0\right)\left(1+\gamma_0\right). \tag{187}$$

Setting the time subscripts to period zero and rearranging equation (97), I obtain the initial condition for taxes, net of transfers,

$$\tilde{t}_0 = \tilde{g}_0-\left(1-\frac{1+r_0}{\left(1+\gamma_0\right)\left(1+n_0\right)}\right)\tilde{b}_0. \tag{188}$$

Setting the time subscripts to period zero and rearranging equation (108), I obtain the initial condition for non-superannuation transfers payments,

$$\tilde{z}_0 = \tilde{t}_0-\left(\tau_0^c\tilde{c}_0+\tau_0^l\tilde{w}_0 h_0 l_0+\tau_0^k\left(R_0-\omega\delta\right)\tilde{k}_0-\left(1-\tau^s\right)\tilde{w}_0 d\Psi_0-\tilde{\mathfrak{e}}_0\right), \tag{189}$$

so that the initial value for non-superannuation transfers to GDP is determined by,

$$\vartheta_{z,0} = \frac{\tilde{z}_0}{p_0 \tilde{y}_0 \mathcal{A}_{z,0}}. \tag{190}$$

The remaining initial conditions for $\mathfrak{t}_0$, $\tilde{pb}_0$, $\tilde{ob}_0$, $\tilde{rb}_0$, $\tilde{t}^k_0$, $\tilde{t}^c_0$, $\tilde{t}^l_0$ and $\tilde{e}_0$ can be obtained using equations (137) to (144), with the time subscripts set to zero,

$$\tilde{\mathfrak{t}}_0 = \tau^c_0 \tilde{c}_0 + \tau^l_0 \tilde{w}_0 h_0 l_0 + \tau^k_0 \left(R_0 - \omega\delta\right) \tilde{k}_0 + \tau^s \tilde{w}_0 d \Psi_0, \tag{191}$$

$$\tilde{pb}_0 = \tilde{t}_0 - \tilde{g}_0, \tag{192}$$

$$\tilde{ob}_0 = \tilde{t}_0 - \tilde{g}_0 - r_0 \tilde{b}_0 \left(1 + \gamma_0\right)^{-1} \left(1 + n_0\right)^{-1}, \tag{193}$$

$$\tilde{rb}_0 = r_0 \tilde{b}_0 \left(1 + \gamma_0\right)^{-1} \left(1 + n_0\right)^{-1}, \tag{194}$$

$$\tilde{t}^k_0 = \tau^k_0 \left(R_0 - \omega\delta\right) \tilde{k}_0, \tag{195}$$

$$\tilde{t}^c_0 = \tau^c_0 \tilde{c}_0, \tag{196}$$

$$\tilde{t}^l_0 = \tau^l_0 \tilde{w}_0 \ell_0. \tag{197}$$

$$\tilde{e}_0 = \tilde{g}_0 + \tilde{s}_0 - \tilde{z}_0 + \tilde{\mathfrak{e}}_0. \tag{198}$$

D. Finding the Terminal Steady State

For most of the projections I run, the terminal steady state is not the same as the initial steady state. Values for some of the exogenous variables change over the transition path, while some of the policy variables need to change to ensure the net debt target is met and the economy is in equilibrium. In this section, I describe how I solve for the terminal steady state. I find values for the 41 endogenous variables: $\tilde{c}_T$, l_T, $\tilde{i}_T$, $\tilde{\tilde{k}}_T$, $\tilde{k}_T$, $\tilde{b}_T$, $\tilde{f}_T$, r_T, R_T, $\tilde{w}_T$, $\tilde{t}_T$, $\tilde{g}_T$, $\tilde{g}_{h,T}$, $\tilde{g}_{i,T}$, $\tilde{g}_{g,T}$, $\tilde{\lambda}_T$, $\tilde{z}_T$, $b^{\star}_T$, $y^*_{H,T}$, $\tilde{y}_T$, q_T, p_T, Γ_T, $\tilde{k}_{G,T}$, Φ_T, Υ_T, $\tilde{nx}_T$, $\tilde{y}_{F,T}$, $\tilde{s}_T$, $\tilde{c}^{\star}_T$, $\tilde{\mathfrak{t}}_T$, $\tilde{pb}_T$, $\tilde{ob}_T$, $\tilde{rb}_T$, $\tilde{t}^k_T$, $\tilde{t}^c_T$, $\tilde{t}^l_T$, $\tilde{e}_T$, $\tau^{l\star}_t$, $\tau^{k\star}_t$, $\tau^{c\star}_t$. In the terminal steady state, I treat the value for labour, l_T, as endogenous. This is crucial, because changes in government policy should impact all endogenous variables in the terminal steady state. This is a key difference to how the initial steady state was solved. It also means there is no "pencil and paper" solution for the steady state model, and that the steady state levels of output and the terms of trade need to be solved numerically.

The exogenous shock processes; $\mathcal{A}_T$, $\mathcal{A}_{\gamma,T}$, $\mathcal{A}_{c,T}$, $\mathcal{A}_{i,T}$, $\mathcal{A}_{q,T}$, $\mathcal{A}_{g_h,T}$, $\mathcal{A}_{g_i,T}$, $\mathcal{A}_{g_g,T}$, $\mathcal{A}_{z,T}$, $\mathcal{A}_{x,T}$, $\mathcal{A}_{m,T}$, $\mathcal{A}_{b,T}$, have the same values in the terminal steady state as they did in the initial steady state.

Capacity utilisation, $\upsilon_T = \upsilon_0 = 1$, EQC payments, $\tilde{\mathfrak{e}}_T = \tilde{\mathfrak{e}}_0 = 0$, the target net foreign asset to GDP ratio, $f^{\star}_T = f^{\star}_0$, the total factor productivity growth rate $\gamma_T = \gamma_0 = \gamma$, the exogenous age-related human capital term, $h_T = h_0 = 1$, the foreign real interest rate, $r^*_T = r^*_0$ and foreign real GDP, $\tilde{y}^*_T = \tilde{y}^*_0$, all take the same values in the terminal steady state as they did in the initial steady state.

Using equations (149) and (150), I obtain terminal conditions for the real interest rate and the risk premium,

$$r_T = \frac{1+\gamma_T}{\beta} - 1, \tag{199}$$

$$\Upsilon_T = \frac{1+r_T}{1+r^*_T}. \tag{200}$$

Upon closer inspection, $r_T = r_0$ and $\Upsilon_T = \Upsilon_0$. Equation (92) implies $\Phi_T = \Phi_0 = 1$ in the terminal steady state.

In the terminal steady state I treat l_T as endogenous and solve for it, while I treat κ_T as exogenous, setting it equal to the same value I found for the initial steady state, $\kappa_T = \kappa_0$.[79] The population growth rate, n_T, is set equal to its date T projected value. In the baseline ageing population scenario, it is assumed that the government maintains the same net debt to GDP target over the entire projection period, so that $b^{\star}_T = b^{\star}_{exog,T} = b^{\star}_{exog,0}$. At the same time, spending pressures mean that $\vartheta_{h,T} > \vartheta_{h,0}$, $\vartheta_{g,T} > \vartheta_{g,0}$, $\vartheta_{z,T} \neq \vartheta_{z,0}$, and an ageing population means that $\Psi_T > \Psi_0$, which increases superannuation spending, assuming government commits to fully funding superannuation for those aged 65+ in period T. In order to ensure the government meets its debt target in period T, government needs to adjust the terminal values of the tax rates, τ^c_T, τ^k_T, τ^l_T and ensure that the government's steady state budget constraint holds. I solve for the average tax rates that ensure the government budget constraint holds in Matlab, using a root finding algorithm.[80]

Because I treat l_T and Γ_T as endogenous in the terminal steady state, I no longer have analytical expressions for output or the terms of trade. This means I need to solve a non-linear system of

79 In the absence of demographic and policy changes, setting κ_T to the same value used in the intial steady state ensures that labour would be the same in both steady states.

80 I use the `fzero` algorithm in Matlab.

simultaneous equations to find terminal conditions for Γ_T and $\tilde{y}_T$. The box below describes the sytem of equations that I solve. This can be solved using Newton's method in Matlab.[81]

Solving a Block of the Terminal Steady State Numerically

Find steady state values of $\tilde{y}_T$ and Γ_T such that,

$$\tilde{\lambda}_T \tilde{w}_T - \varkappa_T \kappa_T (1-\chi)^{-\sigma} \left(\frac{l_T^{\eta}}{1-\tau_T^l} \right) = 0, \tag{201}$$

$$\tilde{y}_T - p_T^{-\varepsilon} \left[\alpha \left(\tilde{c}_T + \tilde{i}_T + \tilde{g}_T \right) + (1-\alpha^*) q_T^{\varepsilon} \tilde{y}_T^* \right] = 0. \tag{202}$$

This is a system of two equations in two unknowns, where the expressions from equations (103), (104), (162) - (165), (145), (153), (176), (167), (178), (161), (169), (170), (171), below, determine the variables, p_T, q_T, $\tilde{g}_T$, $\tilde{i}_T$, l_T, $\tilde{w}_T$, $\tilde{c}_T$, $\tilde{\lambda}_T$, in equations (201) and (202). The terminal relative price level is determined by,

$$p_T = \left[\alpha + (1-\alpha) \Gamma_T \right]^{\frac{1}{\varepsilon-1}}. \tag{203}$$

The terminal real exchange rate is determined by,

$$q_T = \Gamma_T p_T. \tag{204}$$

The terminal level of government expenditure is determined by,

$$\tilde{g}_T = \left(\vartheta_{g_g,T} \mathcal{A}_{g_g,T} + \vartheta_{g_i} \mathcal{A}_{g_i,T} + \vartheta_{g_h,T} \mathcal{A}_{g_h,T} \right) p_T \tilde{y}_T. \tag{205}$$

The terminal terminal level of government investment is determined by,

$$\tilde{g}_{i,T} = \vartheta_{g_i} p_T \tilde{y}_T \mathcal{A}_{g_i,T}. \tag{206}$$

The terminal level of the public capital stock is determined by,

$$\tilde{k}_{G,T} = \tilde{g}_{i,T} \left(1 - \frac{1-\delta_G}{(1+\gamma_T)(1+n_T)} \right)^{-1}. \tag{207}$$

The terminal capital output ratio is determined by,

$$\left(\frac{\tilde{\tilde{k}}}{\tilde{y}} \right)_T = \frac{\beta \left(1-\tau_T^k\right) \theta p_T (1+\gamma_t)(1+n_T)}{\Phi_T (1+\gamma_T) - \beta \omega \tau_T^k \delta - \beta \Phi_T (1-\delta)}. \tag{208}$$

Terminal labour productivity is determined by,

$$\left(\frac{\tilde{y}}{l} \right)_T = h_T \mathcal{A}_T^{\frac{1}{1-\theta}} \left(\left(\frac{\tilde{\tilde{k}}}{\tilde{y}} \right)_T (1+\gamma_T)^{-1} (1+n_T)^{-1} \right)^{\frac{\theta}{1-\theta}} \left(\tilde{k}_{G,T} (1+\gamma_T)^{-1} (1+n_T)^{-1} \right)^{\frac{\theta_G}{1-\theta}}. \tag{209}$$

The terminal investment to GDP ratio is determined by,

$$\left(\frac{\tilde{i}}{\tilde{y}} \right)_T = \left(1 - \frac{1-\delta}{(1+\gamma_T)(1+n_T)} \right) \left(\frac{\tilde{\tilde{k}}}{\tilde{y}} \right)_T. \tag{210}$$

The terminal level of investment is determined by,

$$\tilde{i}_T = \left(\frac{\tilde{i}}{\tilde{y}} \right)_T \tilde{y}_T. \tag{211}$$

[81] I use the `fzolve` algorithm in Matlab.

The terminal level of hours worked is determined by,

$$l_T = \left(\frac{\tilde{y}}{l}\right)_T^{-1} \tilde{y}_T. \tag{212}$$

The terminal real wage level is determined by,

$$\tilde{w}_T = (1-\theta)\, p_T \left(\frac{\tilde{y}}{l}\right)_T h_T^{-1}. \tag{213}$$

The terminal level of net foreign assets is determined by,

$$\tilde{f}_T = 4 \times f_T^{\star} p_T \tilde{y}_T. \tag{214}$$

The terminal consumption level is given by,

$$\tilde{c}_T = p_T\tilde{y}_T - \tilde{g}_T - \tilde{i}_T - \tilde{f}_T \left(1 - \frac{1+r_T}{(1+\gamma_T)(1+n_T)}\right). \tag{215}$$

The terminal level of effective consumption is given by,

$$\tilde{c}_T^{\star} = \tilde{c}_T + \alpha_{g_g}\tilde{g}_{g,T} + \alpha_{g_h}\tilde{g}_{h,T} + \alpha_{g_i}\tilde{g}_{i,T}. \tag{216}$$

The terminal marginal utility of consumption is given by,

$$\tilde{\lambda}_T = \frac{\left(\tilde{c}_T^{\star}(1-\chi)\right)^{-\sigma}}{1+\tau_T^c}. \tag{217}$$

With Γ_T and $\tilde{y}_T$ determined, I can use equations (103) and (104) to find expressions for the relative price level and the real exchange rate,

$$p_T = \left[\alpha + (1-\alpha)\,\Gamma_T^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}}, \tag{218}$$

$$q_T = \Gamma_T p_T. \tag{219}$$

Using (183) I obtain the terminal condition for exports,

$$y_{H,T}^{*} = (1-\alpha^{*})\left(\frac{p_T}{q_T}\right)^{-\varepsilon}\tilde{y}_T^{*}. \tag{220}$$

From (153) the capital output ratio in the terminal steady state is,

$$\left(\frac{\tilde{\tilde{k}}}{\tilde{y}}\right)_T = \frac{\beta\left(1-\tau_T^k\right)\theta p_T (1+\gamma)(1+n_T)}{\Phi_T(1+\gamma_T) - \beta\omega\tau_T^k\delta - \beta\Phi_T(1-\delta)}, \tag{221}$$

which can be combined with the value of $\tilde{y}_T$ that has been found to get the terminal condition for the private capital stock,

$$\tilde{\tilde{k}}_T = \left(\frac{\tilde{\tilde{k}}}{\tilde{y}}\right)_T \tilde{y}_T. \tag{222}$$

Equation (167) can be used to find investment,

$$\tilde{i}_T = \left(1 - \frac{1-\delta}{(1+\gamma)(1+n_T)}\right)\tilde{\tilde{k}}_T. \tag{223}$$

Equations (162) - (165) and (116) can be used to get the components of government expenditure,

$$\tilde{g}_{g,T} = \vartheta_{g_g,T}\, p_T \tilde{y}_T \mathscr{A}_{g_g,T}, \tag{224}$$

$$\tilde{g}_{i,T} = \vartheta_{g_i} p_T \tilde{y}_T \mathcal{A}_{g_i,T}, \tag{225}$$

$$\tilde{g}_{h,T} = \vartheta_{g_h,T} p_T \tilde{y}_T \mathcal{A}_{g_h,T}, \tag{226}$$

$$\tilde{g}_T = \tilde{g}_{i,T} + \tilde{g}_{h,T} + \tilde{g}_{g,T}, \tag{227}$$

$$\tilde{z}_T = \vartheta_{z,T} p_T \tilde{y}_T \mathcal{A}_{z,T}. \tag{228}$$

The public capital stock can be obtained using equation (146),

$$\tilde{k}_{G,T} = \frac{\tilde{g}_{i,T}}{1 - \frac{1-\delta_G}{(1+\gamma_T)(1+n_T)}}. \tag{229}$$

The rental/dividend rate can be obtained using equation (187),

$$R_T = \theta p_T \left(\frac{\tilde{\tilde{k}}}{\tilde{y}}\right)_T^{-1} (1+\gamma_T)(1+n_T). \tag{230}$$

The effective capital stock in the terminal steady state can be determined using equation (94),

$$\tilde{k}_T = \frac{\upsilon_T \tilde{\tilde{k}}_T}{(1+\gamma_T)(1+n_T)}. \tag{231}$$

Labour productivity is determined by equation (176),

$$\left(\frac{\tilde{y}}{l}\right)_T = h_T \mathcal{A}_T^{\frac{1}{1-\theta}} \left(\left(\frac{\tilde{\tilde{k}}}{\tilde{y}}\right)_T (1+\gamma_T)^{-1} (1+n_T)^{-1}\right)^{\frac{\theta}{1-\theta}} \left(\tilde{k}_{G,T} (1+\gamma_T)^{-1} (1+n_T)^{-1}\right)^{\frac{\theta_G}{1-\theta}}, \tag{232}$$

which can be used to find date T labour,

$$l_T = \left(\frac{\tilde{y}}{l}\right)_T^{-1} \tilde{y}_T, \tag{233}$$

and date T real wages when combined with equation (178),

$$\tilde{w}_T = (1-\theta) p_T \left(\frac{\tilde{y}}{l}\right)_T (h_T)^{-1}. \tag{234}$$

The terminal condition for net foreign assets are determined using equation (161),

$$\tilde{f}_T = 4 \times f_T^* p_T \tilde{y}_T. \tag{235}$$

Using equations (169), (170) and (171) I obtain the level of consumption in the terminal steady state,

$$\tilde{c}_T = p_T \tilde{y}_T - \tilde{g}_T - \tilde{i}_T - \tilde{f}_T \left(1 - \frac{1+r_T}{(1+\gamma_T)(1+n_T)}\right), \tag{236}$$

terminal effective consumption,

$$\tilde{c}_T^{\star} = \tilde{c}_T + \alpha_{g_g} \tilde{g}_{g,T} + \alpha_{g_h} \tilde{g}_{h,T} + \alpha_{g_i} \tilde{g}_{i,T}, \tag{237}$$

and the marginal utility of consumption,

$$\tilde{\lambda}_T = \frac{(\tilde{c}_T (1-\chi))^{-\sigma}}{1+\tau_T^c}. \tag{238}$$

I use equation (135) to find the terminal condition for imports,

$$\tilde{y}_{F,T} = (1-\alpha) q_T^{-\varepsilon} \left(\tilde{c}_T + \tilde{i}_T + \tilde{g}_T\right), \tag{239}$$

and equation (173) to find the terminal condition for net exports,

$$\tilde{nx}_T = \left(1 - (1 + r_T)(1 + \gamma_T)^{-1}(1 + n_T)^{-1}\right)\tilde{f}_T. \tag{240}$$

The terminal condition for taxes net of transfers can be found using equation (108),

$$\tilde{t}_T = \tau_T^c \tilde{c}_T + \tau^l \tilde{w}_T h_T l_T + \tau_T^K (R_T - \omega\delta) \tilde{\tilde{\bar{k}}}_t (1 + \gamma)^{-1}(1 + n_T)^{-1} + \tilde{z}_T - (1 - \tau^s)\tilde{w}_T d\Psi_T - \tilde{\mathfrak{e}}_T. \tag{241}$$

Finally the terminal conditions for $\mathfrak{t}_T$, $\tilde{pb}_T$, $\tilde{ob}_T$, $\tilde{rb}_T$, $\tilde{t}_T^k$, $\tilde{t}_T^c$, $\tilde{t}_T^l$ and $\tilde{e}_T$ can be obtained from equations (137) to (144),

$$\tilde{\mathfrak{t}}_T = \tau_T^c \tilde{c}_T + \tau^l \tilde{w}_T h_T l_T + \tau_T^K (R_T - \omega\delta) \tilde{\tilde{\bar{k}}}_t (1 + \gamma)^{-1}(1 + n_T)^{-1} + \tau^s \tilde{w}_T d\Psi_T, \tag{242}$$

$$\tilde{s}_T = \tilde{w}_T d\Psi_T, \tag{243}$$

$$\tilde{pb}_T = \tilde{t}_T - \tilde{g}_T, \tag{244}$$

$$\tilde{ob}_T = \tilde{t}_T - \tilde{g}_T - r_T \tilde{b}_T (1 + \gamma_T)^{-1}(1 + n_T)^{-1}, \tag{245}$$

$$\tilde{rb}_T = r_T \tilde{b}_T (1 + \gamma_T)^{-1}(1 + n_T)^{-1}, \tag{246}$$

$$\tilde{t}_T^k = \tau_T^k (R_T - \omega\delta)\tilde{k}_T, \tag{247}$$

$$\tilde{t}_T^c = \tau_T^c \tilde{c}_T, \tag{248}$$

$$\tilde{t}_T^l = \tau_T^l \tilde{w}_T \ell_T, \tag{249}$$

$$\tilde{e}_T = \tilde{g}_T + \tilde{s}_T - \tilde{z}_T + \tilde{\mathfrak{e}}_T. \tag{250}$$

E. Demographic Wedges

In this section, I describe how the demographic wedges are calculated and incorporated in the model. I take the same approach as Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020) to approximate heterogeneous agents in a representative agent model. Total effective hours worked, ℓ_t, is defined as,

$$\ell_t = \sum_j N_{t,j} h_j \ell_{t,j}, \tag{251}$$

where $N_{t,j}$ is the size of the population of age j at time t, h_j is the average human capital of a person of age j (adjusted for the overall level of productivity in the economy) and $\ell_{t,j}$ is the average proportion of time spent working (hours worked normalised by the total fixed amount that people are able to work) for a person of age j in period t. Date t average hours worked per person are defined as,

$$l_t = \frac{\sum_j N_{t,j} h_j \ell_{t,j}}{\sum_j N_{t,j} h_j}, \tag{252}$$

date t average human capital per person, h_t, is defined as,

$$h_t = \frac{\sum_j N_{t,j} h_j}{\sum_j N_{t,j}}, \tag{253}$$

where the total population, N_t, is defined as,

$$N_t = \sum_j N_{t,j}. \tag{254}$$

Total effective hours worked can then be written as the product of average hours worked per person, average human capital per person and the total population, so that,

$$\ell_t = \sum_j N_{t,j} \frac{\sum_j N_{t,j} h_j}{\sum_j N_{t,j}} \frac{\sum_j N_{t,j} h_j \ell_{t,j}}{\sum_j N_{t,j} h_j} = N_t h_t l_t. \tag{255}$$

Aggregate effective consumption is defined as,

$$C_t^\star = \sum_j N_{t,j} c_{t,j}^\star, \tag{256}$$

where,

$$C_t^\star = C_t + \alpha_{g_g} G_{g,t} + \alpha_{g_i} G_{i,t} + \alpha_{g_h} G_{h,t}, \tag{257}$$

and,

$$c_{t,j}^\star = c_{t,j} + \alpha_{g_g} g_{g,t} + \alpha_{g_i} g_{i,t} + \alpha_{g_h} g_{h,t}, \tag{258}$$

which follows from equation (2), so that,

$$C_t + \alpha_{g_g} G_{g,t} + \alpha_{g_i} G_{i,t} + \alpha_{g_h} G_{h,t} = \sum_j N_{t,j} \left(c_{t,j} + \alpha_{g_g} g_{g,t} + \alpha_{g_i} g_{i,t} + \alpha_{g_h} g_{h,t} \right), \tag{259}$$

where $c_{t,j}^\star$ is effective consumption by households that are j years old. Note that the reference points of government consumption and investment in effective consumption are the same regardless of the age cohort. The aggregate utility or social welfare function is defined as,

$$U_t = \sum_j N_{t,j} \lambda^{i,j} \phi_t^{i,j} \left[\frac{\left(c_{t,j}^\star - \chi \left(1+\gamma_t\right)\left(1+n_t\right) \bar{c}_{t-1}^\star \right)^{1-\sigma} A_t^{\sigma-1}}{1-\sigma} - (1-\chi)^{-\sigma} \kappa_j \frac{\ell_{t,j}^{1+\eta}}{1+\eta} \right], \tag{260}$$

which takes a similar form to equation (1), where $\lambda^{i,j}$ is the welfare weight attached to households of age j, $\phi_t^{i,j}$ is an indicator function that takes the value 1 when agents are alive and 0 when they are no longer alive, χ is the weight on the habit stock, γ_t is the growth rate of total factor productivity, n_t is the population growth rate, $\bar{c}^{\star}_{t-1}$ is the per capita aggregate level of effective consumption from period $t-1$, A_t is total factor productivity, κ_j is the age dependent disutility of working, σ is the inverse of the intertemporal elasticity of substitution, η is the inverse of the Frisch labour supply elasticity. Following Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020), the social planner chooses allocations of consumption $c_{t,j}$ and labour $\ell_{t,j}$ to maximise aggregate social welfare in the current period, subject to the aggregation constraints,

$$\mathscr{L} = \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\left[\frac{\left(c^{\star}_{t,j}-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}-\left(1-\chi\right)^{-\sigma}\kappa_j\frac{\ell_{t,j}^{1+\eta}}{1+\eta}\right]+\dots$$

$$\dots+\Lambda_t\left[C^{\star}_t-\sum_j N_{t,j}c^{\star}_{t,j}\right]+\Omega_t\left[\ell_t-\sum_j N_{t,j}h_j\ell_{t,j}\right]. \quad (261)$$

I obtain the first order condition with respect to consumption,

$$\frac{\partial\mathscr{L}}{\partial c_{t,j}} = N_{t,j}\lambda^{i,j}\phi_t^{i,j}\left(c^{\star}_{t,j}-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}\right)^{-\sigma}A_t^{\sigma-1}-\Lambda_t N_{t,j}=0, \quad (262)$$

and the first order condition with respect to labour,

$$\frac{\partial\mathscr{L}}{\partial \ell_{t,j}} = N_{t,j}\lambda^{i,j}\phi_t^{i,j}\kappa_j\ell_{t,j}^{\eta}-\Omega_t N_{t,j}h_j=0. \quad (263)$$

From the first order conditions I obtain,

$$\lambda^{i,j}\phi_t^{i,j}\left(c^{\star}_{t,j}-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}\right)^{-\sigma}A_t^{\sigma-1}=\Lambda_t, \quad (264)$$

and,

$$\lambda^{i,j}\phi_t^{i,j}\kappa_j\ell_{t,j}^{\eta}=\Omega_t h_j. \quad (265)$$

Rearranging equation (264) for $c^{\star}_{t,j}$,

$$\left(c^{\star}_{t,j}-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}\right)^{-\sigma}=\frac{\Lambda_t A_t^{1-\sigma}}{\lambda^{i,j}\phi_t^{i,j}}, \quad (266)$$

$$c^{\star}_{t,j}-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}=\left(\frac{\Lambda_t A_t^{1-\sigma}}{\lambda^{i,j}\phi_t^{i,j}}\right)^{\frac{-1}{\sigma}}, \quad (267)$$

$$c^{\star}_{t,j}=\left(\frac{\Lambda_t A_t^{1-\sigma}}{\lambda^{i,j}\phi_t^{i,j}}\right)^{\frac{-1}{\sigma}}+\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}. \quad (268)$$

Aggregate effective consumption can then be rewritten as,

$$C^{\star}_t=\sum_j N_{t,j}c^{\star}_{t,j}=\sum_j N_{t,j}\left[\left(\frac{\Lambda_t A_t^{1-\sigma}}{\lambda^{i,j}\phi_t^{i,j}}\right)^{\frac{-1}{\sigma}}+\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}^{\star}_{t-1}\right],$$

$$=\left(\Lambda_t A_t^{1-\sigma}\right)^{\frac{-1}{\sigma}}\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}+\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}^{\star}_{t-1}. \quad (269)$$

Aggregate effective consumption relative to the habit stock can be written as,

$$C^{\star}_t-\chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}^{\star}_{t-1}=\left(\Lambda_t A_t^{1-\sigma}\right)^{\frac{-1}{\sigma}}\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}. \quad (270)$$

Using equation (264) to eliminate the Lagrange multiplier gives,

$$C_t^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^{\star} = \left(c_{t,j}^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}_{t-1}^{\star}\right)\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{-1}{\sigma}}\sum_j N_{t,j},\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}, \tag{271}$$

which then allows cohort effective consumption, relative to the habit stock, to be written as a fraction of aggregate effective consumption, relative to the aggregate habit stock,

$$c_{t,j}^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{c}_{t-1}^{\star} = \frac{\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}{\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}\left(C_t^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^{\star}\right). \tag{272}$$

Rearranging equation (265) for $\ell_{t,j}$,

$$\ell_{t,j}^{\eta} = \frac{\Omega_t h_j}{\lambda^{i,j}\phi_t^{i,j}\kappa_j}, \tag{273}$$

$$\ell_{t,j} = \left(\frac{\Omega_t h_j}{\lambda^{i,j}\phi_t^{i,j}\kappa_j}\right)^{\frac{1}{\eta}}. \tag{274}$$

Aggregate effective labour can be constructed according to,

$$\ell_t = \sum_j N_{t,j}h_j\ell_{t,j} = \sum_j N_{t,j}h_j\left(\frac{\Omega_t h_j}{\lambda^{i,j}\phi_t^{i,j}\kappa_j}\right)^{\frac{1}{\eta}} = \Omega_t^{\frac{1}{\eta}}\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}. \tag{275}$$

Using equation (265) to eliminate the Lagrange multiplier and rearranging for $\ell_{j,t}$,

$$\ell_t = \ell_{t,j}h_j^{\frac{-1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{1}{\eta}}\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}\ell_t, \tag{276}$$

$$\ell_{t,j} = \frac{h_j^{\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}{\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}\ell_t. \tag{277}$$

Plugging the expression for effective consumption relative to the habit stock, (272) and labour (277) into the aggregate utility function, (260), gives,

$$U_t = \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\left[\begin{array}{c}\dfrac{\left(\frac{\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}{\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}\left(C_t^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^{\star}\right)\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma} - \ldots \\ \ldots - \left(1-\chi\right)^{-\sigma}\kappa_j\dfrac{\left(\frac{h_j^{\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}{\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}\ell_t\right)^{1+\eta}}{1+\eta}\end{array}\right]. \tag{278}$$

This can be rewritten in terms of aggregate per capita variables and demographic wedges by rearranging and cancelling terms through the following set of steps,

$$U_t = \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\left[\begin{array}{c}\dfrac{\left(\frac{\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}{\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}\left(C_t^{\star} - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^{\star}\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma} - \ldots \\ \ldots - \left(1-\chi\right)^{-\sigma}\kappa_j\dfrac{\left(\frac{h_j^{\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}{\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}\right)^{1+\eta}\ell_t^{1+\eta}}{1+\eta}\end{array}\right], \tag{279}$$

$$U_t = \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\left(\frac{\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}{\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}\left[\frac{\left(C_t^\star - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^\star\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}\right]-\ldots$$

$$\ldots - \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\kappa_j\left(\frac{h_j^{\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}{\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}}\right)^{1+\eta}\left(1-\chi\right)^{-\sigma}\frac{\ell_t^{1+\eta}}{1+\eta}, \quad (280)$$

$$U_t = \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\frac{\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}-1}}{\left(\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}\right)^{1-\sigma}}\left[\frac{\left(C_t^\star - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^\star\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}\right]-\ldots$$

$$\ldots - \sum_j N_{t,j}\lambda^{i,j}\phi_t^{i,j}\kappa_j\frac{h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{-\left(1+\frac{1}{\eta}\right)}}{\left(\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}\right)^{1+\eta}}\left(1-\chi\right)^{-\sigma}\frac{\ell_t^{1+\eta}}{1+\eta}, \quad (281)$$

$$U_t = \left(\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}\right)^{\sigma}\left[\frac{\left(C_t^\star - \chi\left(1+\gamma_t\right)\left(1+n_t\right)\bar{C}_{t-1}^\star\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}\right]-\ldots$$

$$\ldots - \left(\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}\right)^{-\eta}\left(1-\chi\right)^{-\sigma}\frac{\ell_t^{1+\eta}}{1+\eta}, \quad (282)$$

Substituting in the definitions of the aggregate variables from equations (259) and (255) gives,

$$U_t = \left(\sum_j N_{t,j}\left(\lambda^{i,j}\phi_t^{i,j}\right)^{\frac{1}{\sigma}}\right)^{\sigma}N_t^{1-\sigma}\left[\frac{\left(c_t^\star - \chi\left(1+\gamma_t\right)\bar{c}_{t-1}^\star\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}\right]-\ldots$$

$$\ldots - \left(\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}\right)^{-\eta}N_t^{1+\eta}h_t^{1+\eta}\left(1-\chi\right)^{-\sigma}\frac{l_t^{1+\eta}}{1+\eta}. \quad (283)$$

Following Jones (2018), Papetti (2019) and Lis, Nickel, and Papetti (2020), I set $\lambda^{i,j} = 1$ for all i and j, so that,

$$U_t = N_t\left(\left[\frac{\left(c_t^\star - \chi\left(1+\gamma_t\right)\bar{c}_{t-1}^\star\right)^{1-\sigma}A_t^{\sigma-1}}{1-\sigma}\right] - \varkappa_t h_t\left(1-\chi\right)^{-\sigma}\frac{l_t^{1+\eta}}{1+\eta}\right), \quad (284)$$

where $\varkappa_t$ is defined as follows,

$$\varkappa_t = \left(\sum_j N_{t,j}h_j^{1+\frac{1}{\eta}}\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}\right)^{-\eta}N_t^{\eta}h_t^{\eta}, \quad (285)$$

$$= \left(N_t^{-1}h_t^{-1}\sum_j N_{t,j}h_j\left(\lambda^{i,j}\phi_t^{i,j}\kappa_j\right)^{\frac{-1}{\eta}}h_j^{\frac{1}{\eta}}\right)^{-\eta}, \quad (286)$$

$$= \left(\sum_{j=1}^{J} e_{t,j}\left(\frac{h_j}{\kappa_j}\right)^{\frac{1}{\eta}}\right)^{-\eta}, \quad (287)$$

and $e_{t,j} = \frac{N_{t,j}h_j}{N_t h_t}$. Demographic factors can then be represented by the exogenous wedges, h_t and $\varkappa_t$, where $\varkappa_t$ represents the aggregate disutility wedge that is a function of the population structure.

The aggregate budget constraint in equation (4) can be obtained by summing over all households. Each household of age j has the budget constraint,

$$\begin{aligned}
&(1+\tau_t^c)\, c_{t,j} + i_{t,j} + b_{t,j}\exp(\mathscr{A}_{b,t}) + f_{t,j}\exp(\mathscr{A}_{b,t}) = \dots \\
&\quad \dots (1+r_{t-1})\, b_{t-1,j} + (1+r_{t-1})\, f_{t-1,j} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\, \bar{k}_{t-1,j} - \dots \\
&\quad \dots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\, \bar{k}_{t-1,j} + \left(1-\tau_t^l\right) w_t h_j \ell_{t,j} + \dots \\
&\quad + \dots \omega \tau_t^k \delta (1-\varphi\Delta_t)\, \bar{k}_{t-1,j} + (1-\tau^s)\, w_t d_{t,j} - z_{t,j} + \mathfrak{e}_{t,j} + \xi_{t,j}, \qquad (288)
\end{aligned}$$

multiplying by $N_{t,j}$, I obtain,

$$\begin{aligned}
&(1+\tau_t^c)\, N_{t,j}c_{t,j} + N_{t,j}i_{t,j} + N_{t,j}b_{t,j}\exp(\mathscr{A}_{b,t}) + N_{t,j}f_{t,j}\exp(\mathscr{A}_{b,t}) = \dots \\
&\quad \dots (1+r_{t-1})\, N_{t,j}b_{t-1,j} + (1+r_{t-1})\, N_{t,j}f_{t-1,j} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\, N_{t,j}\bar{k}_{t-1,j} - \dots \\
&\quad \dots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\, N_{t,j}\bar{k}_{t-1,j} + \left(1-\tau_t^l\right) w_t N_{t,j} h_j \ell_{t,j} + \dots \\
&\quad + \dots \omega \tau_t^k \delta (1-\varphi\Delta_t)\, N_{t,j}\bar{k}_{t-1,j} + (1-\tau^s)\, w_t N_{t,j}d_{t,j} - N_{t,j}z_{t,j} + N_{t,j}\mathfrak{e}_{t,j} + N_{t,j}\xi_{t,j}, \qquad (289)
\end{aligned}$$

summing over all agents I obtain,

$$\begin{aligned}
&(1+\tau_t^c)\sum_{j=1}^J N_{t,j}c_{t,j} + \sum_{j=1}^J N_{t,j}i_{t,j} + \sum_{j=1}^J N_{t,j}b_{t,j}\exp(\mathscr{A}_{b,t}) + \sum_{j=1}^J N_{t,j}f_{t,j}\exp(\mathscr{A}_{b,t}) = \dots \\
&\dots (1+r_{t-1})\sum_{j=1}^J N_{t,j}b_{t-1,j} + (1+r_{t-1})\sum_{j=1}^J N_{t,j}f_{t-1,j} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\sum_{j=1}^J N_{t,j}\bar{k}_{t-1,j} - \dots \\
&\dots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\sum_{j=1}^J N_{t,j}\bar{k}_{t-1,j} + \left(1-\tau_t^l\right) w_t \sum_{j=1}^J N_{t,j}h_j\ell_{t,j} + \dots \\
&+ \dots \omega \tau_t^k \delta (1-\varphi\Delta_t)\sum_{j=1}^J N_{t,j}\bar{k}_{t-1,j} + (1-\tau^s)\, w_t \sum_{j=1}^J N_{t,j}d_{t,j} - \sum_{j=1}^J N_{t,j}z_{t,j} + \sum_{j=1}^J N_{t,j}\mathfrak{e}_{t,j} + \sum_{j=1}^J N_{t,j}\xi_{t,j}. \qquad (290)
\end{aligned}$$

Making use of $X_t = \sum_{j=1}^J N_{t,j}x_{t,j}$ for C_t, I_t, B_t, F_t, $\bar{K}_t$, Z_t, $\mathfrak{E}_t$, Ξ_t and $c_{t,j}$, $i_{t,j}$, $b_{t,j}$, $f_{t,j}$, $\bar{k}_{t,j}$, $z_{t,j}$, $\mathfrak{e}_{t,j}$, $\xi_{t,j}$, I can rewrite equation (290) as,

$$\begin{aligned}
&(1+\tau_t^c)\, C_t + I_t + B_t\exp(\mathscr{A}_{b,t}) + F_t\exp(\mathscr{A}_{b,t}) = \dots \\
&\quad \dots (1+r_{t-1})\, B_{t-1} + (1+r_{t-1})\, F_{t-1} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\, \bar{K}_{t-1} - \dots \\
&\quad \dots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\, \bar{K}_{t-1} + \left(1-\tau_t^l\right) w_t \ell_t + \dots \\
&\quad + \dots \omega \tau_t^k \delta (1-\varphi\Delta_t)\, \bar{K}_{t-1} + (1-\tau^s)\, w_t d N_{r,t} - Z_t + \mathfrak{E}_t + \Xi_t. \qquad (291)
\end{aligned}$$

Replacing ℓ_t with $N_t h_t l_t$ gives equation (4),

$$\begin{aligned}
&(1+\tau_t^c)\, C_t + I_t + B_t\exp(\mathscr{A}_{b,t}) + F_t\exp(\mathscr{A}_{b,t}) = \dots \\
&\quad \dots (1+r_{t-1})\, B_{t-1} + (1+r_{t-1})\, F_{t-1} + \left(1-\tau_t^k\right) R_t \upsilon_t (1-\varphi\Delta_t)\, \bar{K}_{t-1} - \dots \\
&\quad \dots - \left(\gamma_{\upsilon,1}(\upsilon_1 - 1) + \frac{\gamma_{\upsilon,2}}{2}(\upsilon_t - 1)^2\right)(1-\varphi\Delta_t)\, \bar{K}_{t-1} + \left(1-\tau_t^l\right) w_t N_t h_t l_t + \dots \\
&\quad + \dots \omega \tau_t^k \delta (1-\varphi\Delta_t)\, \bar{K}_{t-1} + (1-\tau^s)\, w_t d N_{r,t} - Z_t + \mathfrak{E}_t + \Xi_t. \qquad (292)
\end{aligned}$$

F. Calibration

I choose the model parameters using a process known as calibration. Gomme and Rupert (2007) describe calibration as choosing functional forms for the utility and production functions, and assigning values to the parameters of the model based on either micro-evidence or long-run growth facts. The description, definition and methodology of calibration is quite vague, with some considering it ad hoc, but this is because calibration is the name of a collection of methods for choosing model parameters. The three main methods for "pinning down" parameters used by calibrationists/calibrators include:

- Choosing parameters to match the model's moments to their counterparts in the data. Calibration targets are usually first moments or great ratios, but can be anything (higher order moments, variances, correlations, impulse response functions or even other models). Assessments of fit are usually made based on an arbitrary judgement of what is close.
- Taking parameters from empirical micro studies. This practise is less frequently used due to potential issues comparing micro estimates with macro estimates.
- Taking parameters from the literature/other studies. While frowned upon by some (see Cooley, 1997), this has become an extremely common practice.[82]

In this appendix, I describe how the parameters were chosen for the baseline ageing population scenario. The NCGM is primarily calibrated by matching great ratios, averages from the data and taking parameters from the literature. The growth rate of labour-augmenting technology, γ is chosen to be 0.0025, so that the annual growth rate of total factor productivity is 1 percent matching the labour productivity growth rate used in the LTFM. The population growth rate, n is chosen to be 0.003 for the initial steady state, the same value that I use for the first 20 periods of the projection and n is set equal to 0.0011 in the terminal steady state, consistent with the population projections. The discount factor is determined according to $\beta = \frac{1+\gamma}{1+r_t}$ where r_t is the average real interest rate in New Zealand between 1994 and 2018.

The weight on the habit stock, χ, is set to 0.8, which is roughly in the middle of the range for New Zealand estimates. Santacreu (2005) estimates the weight on the habit stock to be 0.946, Liu (2006) finds 0.924, while Kamber et al. (2015) estimate a weight of 0.5. I set the inverse of the intertemporal elasticity of substitution, σ, to 1. Typical values for this parameter range between 1 and 5. While determining the sensitivity of consumption to the interest rate, this parameter also has implications for the size of the income/wealth effect (how changes in government spending affect labour supply by altering the marginal utility of consumption and the benefits of working). Larger values result in a larger income/wealth effect and less negative, or even positive permanent balanced budget government consumption multipliers. So as not to overstate the income/wealth effect I set this parameter at the bottom of the range, consistent with Turnovsky and Chatterjee (2002) and Turnovsky (2004).

I set the inverse of the Frisch elasticity of labour supply, η, to 2, implying a labour supply elasticity of 0.5. Work by Creedy and Mok (2017) shows that labour supply elasticities in New Zealand vary from 0.06 in single men under the age of 30, through to 1.29 in sole parents over the age of 60. After reviewing a number of macro and microeconomic studies Chetty et al. (2011) suggest a labour supply elasticity of 0.75 for use in macro models. Leeper, Plante, and Traum (2010) and Leeper, Walker, and Yang (2010), both fiscal studies (using US data), use 0.5 for their prior and estimate values in the neighbourhood of 0.5. Erceg and Lindé (2013) use 0.5 stating that

[82] Cooley (1997) objects to using parameters from other studies because the purpose of the research can often dictate parameter choices. Parameter choices that may be suitable for one context may be unsuitable for other contexts.

it makes sense for tax and fiscal work in macro models, being a good comprimise between traditional macro and micro estimates. The CBO uses 0.4 in their life cycle growth model, which they use to investigate tax and fiscal policy issues (see CBO, 2012) and the IMF use 0.5 in GIMF, a DSGE model they use for monetary and fiscal policy analysis (see Kumhof et al., 2010).[83]

Capital's share of income, θ, is set to 0.3, following Kehoe and Ruhl (2003). Smith and Thoenissen (2018) estimate a capital share of 0.33 for New Zealand. The productivity commission did work on the measured economy (approximately 60 percent of the total economy) and estimated a capital share of income equal to 0.45 (see Conway, Meehan, and Parham, 2015). I favour a lower value of θ as it is more consistent with the great ratios.

I set the elasticity of output with respect to government capital, θ_G, to 0.07. This parameter typically ranges from 0 to 0.1. Following Leeper, Walker, and Yang (2010), I chose this parameter so that the marginal product of public capital is slightly smaller than the marginal product of private capital. If the marginal product of public capital were higher than the marginal product of private capital, this would suggest the public sector is more productive than the private sector, and that the private sector would have an incentive to carry out public investment on their own accord.

I set the home bias parameter, α, to 0.71 which implies a steady state marginal propensity of consuming imports of 0.29 and a steady state share of imports of 29 percent. I set the elasticity of substitution between imported and domestically produced intermediate goods to 1.5, which implies that these goods are weak substitutes. Backus, Kehoe, and Kydland (1993) and Corsetti, Dedola, and Leduc (2008) use the same value for this parameter.

I use a depreciation rate, δ, of 0.0127, implying a 5 percent annual depreciation rate. I set the depreciation rate on public capital, δ_G, equal to the depreciation rate on private capital. The weight on investment adjustment costs, ϕ, is set to 10 which is only slightly larger than the 9.51 value estimated by Fernández-Villaverde (2010). I switch off variable capacity utilisation, because it is a little too sensitive on the transition path.

I can choose the average tax rate on labour income, τ^l, and the average tax rate on consumption expenditure, τ^c freely to calibrate the initial tax ratios (tax revenue to GDP) as they do not affect any of the other great ratios. The consumption tax rate, τ^c, is set to 0.15 which matches the average ratio of nominal consumption tax revenue to consumption over the 1994-2018 period. This is also the current GST tax rate. The average labour tax rate, τ^ℓ, is set to 0.193 to match the average share of labour tax revenue to GDP between 1994 and 2018. The OECD's calculations for the average tax wedge for a single worker in New Zealand between 2000 and 2020 range from 15.9 percent in 2011 up to 21.1 percent in 2007, with an average rate of 18.6 percent and a rate of 19.1 percent in 2020 (see OECD, 2021), which is broadly consistent with my calibration. My calibration is also consistent with the average tax rate faced by someone with earning the average weekly wage or salary in New Zealand in 2021, of $1,247 ($64,844 when annualised), which would be 0.2063.[84]

The choice for the average tax rate on capital income, τ^k, is more difficult and has consequences for both the investment to GDP ratio and the capital tax revenue to GDP ratio. Capital income in the model refers to profits, dividends and rents. While the tax rate on profits and rents is known, the tax rate on dividends varies and depends on shareholders' marginal personal tax rate. The company tax rate is currently 28 percent in New Zealand, although New Zealand has dividend imputation, which means that private share holders are taxed at their marginal tax rate. The highest marginal tax rate in New Zealand is 38 percent and many shareholders will fall into

83 GIMF is an acronym for the Global Integrated Monetary and Fiscal Model.

84 This is the average weekly earnings from main wage or salary in New Zealand at the time of publication.

this and the 33 percent top marginal tax bracket. I set τ^k to 0.3 in the initial steady state which seems to fit the model and the data reasonably well. Coleman (2019) summarizes a number of different tax rates on capital income and wealth for New Zealand which are broadly consistent with my choice. Choosing τ^k has consequences for the ratio of investment to GDP and the capital tax revenue to GDP ratio. Lowering τ^k would match the capital tax ratio better, while raising it would better match the investment to GDP ratio.

The tax responses to deviations of net debt to target, ψ_{τ^c}, ψ_{τ^k} and ψ_{τ^l}, are all set to 0.5, to ensure the budget is approximately balanced in each period. Because of the time-varying target tax rates, these will not have the same interpretation as tax response coefficients used in modelling exercises where the model is not on the transition path.

The elasticity of net foreign debt in the risk premium, ϕ_F, is set to 0.01. A small value is typically used for this parameter, so that net foreign debt dynamics will not have a large impact on the rest of the model. I set the home bias parameter in the foreign country, α^*, to 0.71 for symmetry with the domestic economy.[85] The export adjustment cost, ϕ_X, is set to 2, to ensure that exports aren't too responsive to the exchange rate in the short run.

I do not list the autoregressive coefficients on the shock responses or the shock standard deviations, because these change from scenario to scenario, depending on the modelling exercise. I also change some of the policiy response coefficients for some of the scenarios, to better fit the modelling task at hand.

The full set of calibrated parameters can be found in Table 1.

85 The choice of the home bias parameter does not make any difference to the results given the way foreign GDP is set.

Table 1 — Calibrated Parameters

Parameter	Parameter Definition	Parameter Value
n	Quarterly population growth rate	0.0030
γ	Quarterly total factor productivity growth rate	0.0025
β	Discount factor	0.9944
σ	Inverse of the intertemporal elasticity of substitution	1.0000
η	Inverse of the Frisch elasticity of labour supply	2.0000
χ	Degree of habit indexation	0.8000
α_{g_g}	Determines complementarity/substitutability between $g_{g,t}$ and c_t	0.0000
α_{g_i}	Determines complementarity/substitutability between $g_{i,t}$ and c_t	0.0000
α_{g_h}	Determines complementarity/substitutability between $g_{h,t}$ and c_t	0.0000
δ	Quarterly depreciation rate for private capital	0.0127
θ	Private capital's share of income	0.3000
α	Home bias parameter (import's share of final expenditure)	0.7100
α^*	Home bias parameter in the foreign country	0.7100
r^*	Steady state real interest rate in the foreign country	0.0015
ω	Proportion of depreciation rebated to households	0.6750
δ_G	Quarterly depreciation rate for public capital	0.0127
$\vartheta_{g_g,0}$	General government spending to GDP ratio	0.0889
ϑ_{g_i}	Government investment to GDP ratio	0.0511
$\vartheta_{g_h,0}$	Health spending to GDP ratio	0.0600
τ^k	Tax rate on capital income	0.3000
τ^ℓ	Tax rate on labour income	0.1929
τ^c	Tax rate on capital income	0.1500
$\frac{\tilde{f}}{4\times p\tilde{y}}$	Steady state net foreign asset to GDP ratio	-0.7000
θ_G	Elasticity of output with respect to public capital	0.0700
ℓ_0	Steady state share of time spent working	0.2202
τ^s	Tax rate on government pension income	0.1600
ε	Elasticity of substitution between domestic and foreign goods	1.5000
ϕ	Adjustment cost on investment	10.0000
ϕ_F	External risk premium debt elasticity	0.0100
ϕ_X	Export adjustment cost	2.0000
ψ_{τ_c}	Consumption tax reaction to the debt to GDP target	0.5000
ψ_{τ_k}	Capital tax reaction to the debt to GDP target	0.5000
ψ_{τ_ℓ}	Labour tax reaction to the debt to GDP target	0.5000

Some of the great ratios and their model counterparts are presented in Table 2. A full description of the data used in the calibration can be found in Appendix H. These will form the initial conditions/initial steady state of the model.

Table 2 — Great Ratios and Other Calibration Targets

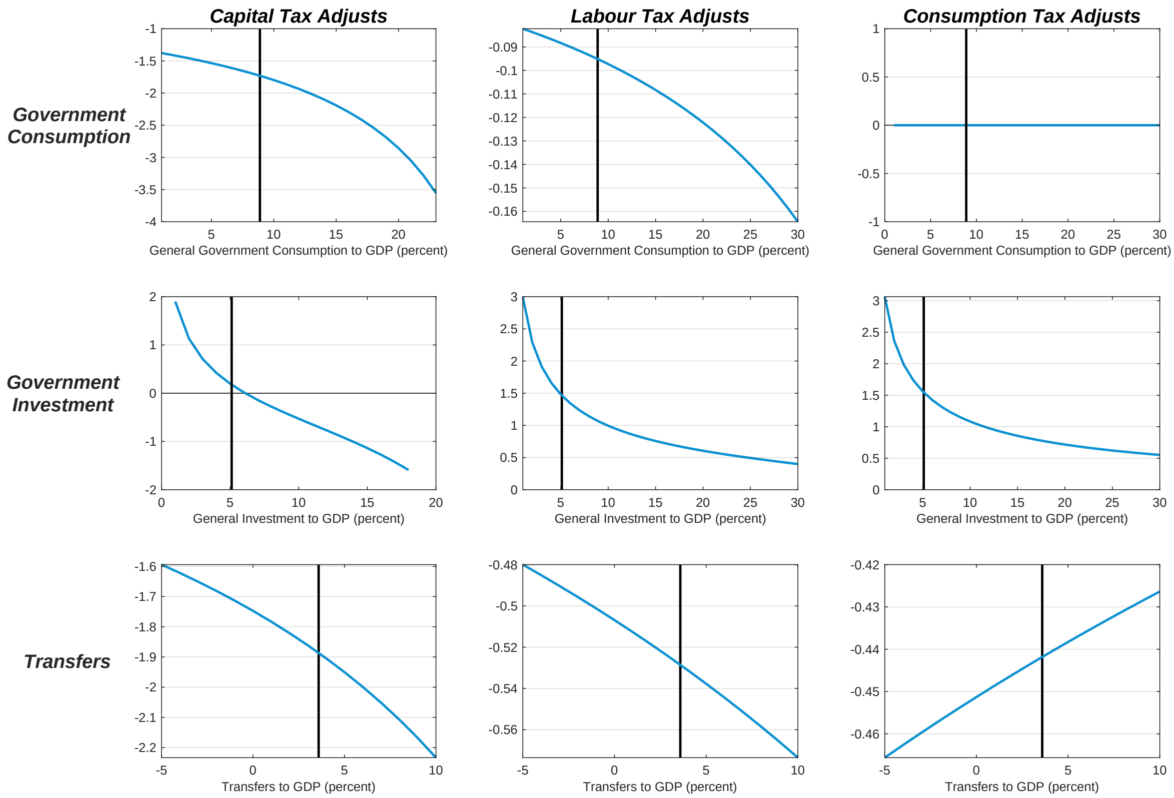
Ratio	Description	Nominal Ratio	Real Ratio	Model SS	LTFM
C/Y	Average consumption to GDP ratio	0.5793	0.5790	0.5836	
I/Y	Average private investment to GDP ratio	0.1706	0.1681	0.2090	
G/Y	Average total govt. spending to GDP ratio	0.2310	0.2411	0.2000	
G_I/Y	Average govt. investment to GDP ratio	0.0511	0.0517	0.0511	
G_H/Y	Average health spending to GDP ratio			0.0600	0.0600
G_G/Y	Average general govt. spending to GDP ratio			0.0889	
X/Y	Average exports to GDP ratio	0.2977	0.2847	0.2953	
M/Y	Average imports to GDP ratio	0.2855	0.2790	0.2878	
T_L/Y	Average labour tax revenue to GDP ratio	0.1350		0.1350	0.1130
T_K/Y	Average capital tax revenue to GDP ratio	0.0563		0.0604	0.0430
T_C/Y	Average consumption tax revenue to GDP ratio	0.0865		0.0875	0.0700
$\mathfrak{T}/Y$	Average total tax revenue to GDP ratio	0.3068		0.2910	0.2750
S/Y	Superannuation spending to GDP ratio			0.0500	0.0490
Z/Y	Other transfers spending to GDP ratio			0.0359	
E/Y	Average total primary spending to GDP ratio			0.2859	
B/Y	Net debt to GDP ratio, target			0.4800	
r	Annualised domestic real interest rate			3.2677	

I match most of the great ratios quite closely. However, the ratio of private investment to GDP is a bit higher in the model than both data measures would suggest. Likewise, the ratio of capital income tax revenue to GDP is slightly higher than the data and the LTFM would suggest. A lower depreciation rate on private investment could lower the investment to GDP ratio in the model, but at 5 percent in annualised terms it is probably at the bottom of an acceptable range. Cutting the tax rate on capital income would better match the capital tax revenue to GDP ratio, however this would push the investment to GDP ratio up even higher. The difficulty in matching the investment to GDP ratio could be due to the absence of land from the production function. The total government spending to GDP ratio is slightly lower than the data would suggest. This is in part due to the ratio of private investment to GDP being a bit on the high side and trying to match the consumption to GDP ratio. It is also due to the discrepancy between national accounts government spending and Crown accounts non-transfers government spending. While I was trying to broadly match national accounts data concepts, a lower government spending to GDP ratio in the model reduces the discrepancy between the inital conditions for government spending in the NCGM and the LTFM.

G. Permanent Fiscal Multipliers and Semi-Elasticitites

In this appendix I calculate the permanent balanced budget government expenditure multipliers and semi-elasticities. These will depend on the model, the model's parameterisation, the type of government expenditure and the specific tax type that adjusts to balance the budget. I compare the model properties with similar models and results from the literature. I calculate average expenditure multipliers, which include marginal multipliers, and plot them in Figure 27.

Figure 27 — Permanent Government Expenditure Multipliers



The black vertical line indicates the calibration of the spending to GDP ratios at the initial steady state. The blue line indicates the average permanent multiplier when moving from the initial steady state to a new steady state ratio when financed through a particular tax type. For example the top left graph shows that if the ratio of general government consumption to GDP permanently increases from the initial calibration of 9 percent to 23 percent and this is financed by increases in capital tax rates, the average permanent mulitiplier would be -3.55. This would suggest a permanent $1 increase in government consumption would lead to a $3.55 permanent reduction in GDP, when financed through increases in capital income tax rates. The marginal multiplier will be the intersection of the vertical black line and the blue line. The graphs show the non-linearity and asymmetry in the permanent average fiscal multipliers.

Marginal multipliers are colated in Table 3

Table 3 — Marginal Multipliers

Expenditure	Capital Tax Adjusts	Labour Tax Adjusts	Consumption Tax Adjusts
Government Consumption	-1.7357	-0.0951	0
Government Investment	0.1733	1.4645	1.5449
Transfers	-1.8830	-0.5286	-0.4418

The semi-elasticities are presented in Table 4.

Table 4 – Semi-Elasticities

Expenditure	Capital Tax Adjusts	Labour Tax Adjusts	Consumption Tax Adjusts
Government Consumption	-1.0025	-0.0629	0
Government Investment	0.1166	1.0554	1.1183
Transfers	-1.1759	-0.3458	-0.2900

The calibration of the inverse of the intertemporal elasticity of substitution (IIES) plays an important role in determining the size of the negative wealth effect from permanent increases in government consumption and government investment. A larger IIES increases the size of the negative wealth effect and the positive labour supply effect from permanently higher government spending on real goods and services. This can have implications for the sign and the size of the permanent balanced budget government spending multipliers. When calibrating the model I set the IIES at 1, which is the bottom of the usual range for calibration to weaken the negative wealth effect, so as not to overstate the impacts of increased government consumption.

From Table 3 the permanent balanced budget government consumption multipliers are all negative, except when consumption taxes adjust and the multiplier is equal to zero. This is because when the IIES is set to 1, the disincentivising effects on labour from higher consumption taxes are exactly offset by the positive supply side effects from the negative wealth effects due to higher government consumption. The permanent balanced budget transfers multipliers are all negative, regardless of how they are funded, although funding them through increases in capital taxation is more costly than increases in labour or consumption taxation. Because government investment is productive, the permanent balanced budget multipliers are all positive, and when funded through increases in either labour or consumption tax are greater than 1.

I can compare the permanent balanced budget multipliers from the NCGM with similar models in the literature. There have been a number of efforts to try and calculate the long-run permanent government spending multipliers. de Walque et al. (2015) survey the core models in use by the central banks that make up the Euro area and find that the long-run fiscal multipliers range from -0.06 to -1.64, with an average of -0.76, for permanent government spending increases funded through increases in labour taxation. While the NCGM fits within this range, it is at the weaker end at -0.10. This is in part due to the lower average labour tax rate in New Zealand compared with the Euro area countries (0.19 versus an OECD average of 0.36) which puts New Zealand in a flatter section of the curve where the negative wealth effect from increases in government expenditure still provides a sizeable offset to the negative effects from increased labour taxation. At higher tax rates the negative effects from increases in labour taxes dominate the negative wealth effect. The choice of the labour supply elasticity could also be playing a role. I have opted to use 0.5, which is typical in tax work using macroeconomic models. The central bank models are business cycles models which typically use larger labour supply elasticities.[86]

When non-distortionary taxation is used to finance permanent increases in government spending, the multipliers increase in size and are positive. de Walque et al. (2015) find long-run fiscal multipliers that range between 0.24 and 1.05, with an average of 0.59, for government spending increases funded through lump-sum taxes (comparable with transfers in the NCGM). Cogan et al. (2010) calculate the long-run government spending multiplier in the Smets and Wouters (2003) model when government spending is financed through lump sum taxes and find a value of 0.4. For plausible calibrations of a simple neoclassical growth model Baxter and King (1993) find

86 Raising the average labour tax rate to 0.36 (the OECD average) in the NCGM and raising the labour supply elasticity to 3 gives permanent government spending multipliers in the neighbourhood of the average of the Euro area models.

long-run government spending multipliers, financed by lump-sum taxes, that range between 0.49 and 1.37. In a similar model calibrated to match the US, Aiyagari, Christiano, and Eichenbaum (1992) find a long-run government spending multiplier of 1.22. Using a Neoclassical growth model calibrated for the US, Turnovsky and Chatterjee (2002) calculate a long-run fiscal multiplier for non-productive government spending, financed by non-distortionary lump-sum taxes of 1.34. They calculate a long-run multiplier of 3.61 for productive government spending financed by non-distortionary lump-sum taxes. When they calculate a long-run multiplier for the optimal permanent increase in a combined productive/non-productive government spending package, financed by distortionary taxation, they get 2.07.

In a similar model Turnovsky (2004) calculates long-run multipliers for productive and non-productive government spending, financed through lump-sum transfers of 5.94 and 1.24 respectively. When non-productive spending is financed through consumption taxes, the multiplier is 0, which is the same result I obtain with the NCGM in this paper. When non-productive government spending is financed through labour taxes the multiplier is -0.89 and when it is financed through capital taxes, it is -5.2. When productive government spending is financed through consumption taxes, the multiplier is 5.28, when it is financed through labour taxes it is 4.88, and when it is financed through capital taxes, it is 3.89. The multipliers in Turnovsky and Chatterjee (2002) and Turnovsky (2004) are larger for a number of reasons. In particular they use an output elasticity to productive government spending of 0.2, which is much higher than the 0 to 0.1 range the literature has now settled upon, and higher than the 0.07 used in this paper. Their labour supply elasticity is 1.1, which is higher than the 0.5 I use, which is more common in tax and fiscal work.

I can also compare the semi-elasticities and multipliers from the NCGM against empirical work. Using a time-series approach that tries to capture the implications of the government budget constraint, Gemmell, Kneller, and Sanz (2011) find that, on average in the OECD, a 1 percentage point increase in the ratio of distortionary taxation revenue to GDP, accompanied by a 1 percentage point increase in the ratio of productive government spending to GDP results in a 0.5 percent reduction in the level of GDP, 20 years after the policy intervention.[87] The semi-elasticities from the NCGM in Table 4 suggest that a permanent 1 percentage point increase in government consumption and transfers spending as a share of GDP, when funded through permanent increases in capital taxation, would result in a 1 percent and a 1.18 percent reduction in the level of GDP, respectively. Likewise, a permanent 1 percentage point increase in the share of government consumption and transfers spending would result in 0.06 percent and a 0.35 percent reduction in GDP respectively. While the results from capital tax financed increases in government consumption are larger than the Gemmell, Kneller, and Sanz (2011) result, the labour tax financed result is smaller, so that it would be possible to find some weighting of the two NCGM results that is consistent with the Gemmell, Kneller, and Sanz (2011) result.[88] From Section 5.1, the total spending semi-elasticity is -0.41 percent, which is lower than Gemmell, Kneller, and Sanz (2011), but also partially financed through consumption taxation, which is not included in their calculations and has a semi-elasticity of 0 in the NCGM.

87 Gemmell, Kneller, and Sanz (2011) define distortionary taxation as taxation on income and profit, social security contributions, taxation on payroll and manpower and taxation on property. Taxation on domestic goods and services is considered non-distortionary. Productive expenditures are general public services expenditure, defense expenditure, educational expenditure, housing expenditure and transport and communications expenditure. Unproductive expenditure includes social security and welfare expenditure, expenditure on recreation and expenditure on economic services.

88 Interpolating the growth results from Alinaghi and Reed (2021) and Gemmell, Kneller, and Sanz (2014) gives GDP reductions in a similar neighbourhood to the NCGM's level results.

H. Data

In this section I describe the data used in the calibration of the model. The tax and total non-transfers government spending definitions used for calculating the great ratios are the same as the data definitions for Figure 14.

H.1 Raw Data Definitions

Table 5 — Raw Data Definitions

Treasury Mnemonic	Description	Data Bank
R90D11AMQ	New Zealand Bank Bill Yields 90-Days (TSY Quarterly Average)	RAT
PCPIQ	Consumer Price Index all groups	PRI
LHEMPZQ	Persons employed in the labour force	LAB
LHHWZQ	Total economy wide hours worked per week	LAB
NGDP_ZQ	Gross Domestic Product - Expenditure measure 09/10 prices	NAT
NCP_ZQ	Final Consumption Expenditure - Private Non Profit Organisations and Households Combined 09/10 prices	NAT
NIP_ZQ	Gross Fixed Capital Formation - Total Market and Non-Market - Private 09/10 prices	NAT
NCGC_ZQ	Final Consumption Expenditure - Central Government 09/10 prices	NAT
NIG_ZQ	Gross Fixed Capital Formation - Total Market and Non-Market - General Government 09/10 prices	NAT
NX_ZQ	Exports of Goods and Services 09/10 prices	NAT
NM_ZQ	Imports of Goods and Services 09/10 prices	NAT
NGDPZQ	Total - Gross Domestic Product - Expenditure measure	NAT
NCPZQ	Final Consumption Expenditure - Private Non-Profit Organisations and Households Combined	NAT
NIPZQ	Gross Fixed Capital Formation - Total Market and Non-Market - Private	NAT
NCGCZQ	Final Consumption Expenditure - Central Government	NAT
NIGZQ	Gross Fixed Capital Formation - Total Market and Non-Market - General Government	NAT
NXZQ	Exports of Goods and Services	NAT
NMZQ	Imports of Goods and Services	NAT
FTTTM	Total Tax Revenue	FIS
FTCPM	Corporate Tax Revenue	FIS
FTRWTM	Tax Revenue Residents' Interest Income	FIS
FTTDWTM	Tax Revenue Dividends	FIS
FTGSTQ	GST Revenue	FIS
FTINDVLM	Individual Tax Revenue	FIS
RTWI11AMQ	New Zealand: Trade-Weighted Exchange Rate Index	RAT
TIINQ	Net International Investment Position	TRA

H.2 Data Used in the Calibration for Matching Steady States

I denote sample averages using a bar over the variable in question. Where the variable is a ratio, the bar over the ratio represents the sample average of the ratio.

Average Real Interest Rate: $r = \bar{r}_t$ where,

$$r_t = \mathrm{i}_t - \pi_t^e,$$

and $\mathrm{i}_t = \text{R90D11AMQ}/400$, π_t^e is the trend from the Hodrick Prescott filtered data π_t, where $\pi_t = \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} - 1$ and $\mathcal{P}_t = \text{PCPIQ}$. I use the Hodrick Prescott filtered CPI inflation trend as a proxy for inflation expectations so that I get a relatively smooth real interest rate estimate.

Average Consumption to GDP Ratio (Real): $\frac{C}{Y}$ where,

$$\frac{C}{Y} = \overline{\frac{\mathcal{C}_t}{\mathcal{Y}_t}},$$

and $\mathcal{C}_t = \text{NCP\_ZQ}$ and $\mathcal{Y}_t = \text{NGDP\_ZQ}$.

Average Investment to GDP Ratio (Real): $\frac{I}{Y}$ where,

$$\frac{I}{Y} = \overline{\frac{\mathcal{I}_t}{\mathcal{Y}_t}},$$

and $\mathcal{I}_t = \text{NIP\_ZQ}$ and $\mathcal{Y}_t = \text{NGDP\_ZQ}$.

Average Government to GDP Ratio (Real): $\frac{G}{Y}$ where,

$$\frac{G}{Y} = \frac{\overline{\mathcal{G}_t}}{\mathcal{Y}_t},$$

and $\mathcal{G}_t$ = NCGC_ZQ + NIG_ZQ and $\mathcal{Y}_t$ = NGDP_ZQ.

Average Exports to GDP Ratio (Real): $\frac{X}{Y}$ where,

$$\frac{X}{Y} = \frac{\overline{\mathcal{X}_t}}{\mathcal{Y}_t},$$

and $\mathcal{X}_t$ = NX_ZQ and $\mathcal{Y}_t$ = NGDP_ZQ.

Average Imports to GDP Ratio (Real): $\frac{M}{Y}$ where,

$$\frac{M}{Y} = \frac{\overline{\mathcal{M}_t}}{\mathcal{Y}_t},$$

and $\mathcal{M}_t$ = NM_ZQ and $\mathcal{Y}_t$ = NGDP_ZQ.

Average Consumption to GDP Ratio (Nominal): $\frac{C}{Y}$ where,

$$\frac{C}{Y} = \frac{\overline{\mathcal{PC}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PC}_t$ = NCPZQ and $\mathcal{PY}_t$ = NGDPZQ.

Average Investment to GDP Ratio (Nominal): $\frac{I}{Y}$ where,

$$\frac{I}{Y} = \frac{\overline{\mathcal{PI}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PI}_t$ = NIPZQ and $\mathcal{PY}_t$ = NGDPZQ.

Average Government to GDP Ratio (Nominal): $\frac{G}{Y}$ where,

$$\frac{G}{Y} = \frac{\overline{\mathcal{PG}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PG}_t$ = NCGCZQ + NIGZQ and $\mathcal{PY}_t$ = NGDPZQ.

Average Exports to GDP Ratio (Nominal): $\frac{X}{Y}$ where,

$$\frac{X}{Y} = \frac{\overline{\mathcal{PX}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PX}_t$ = NXZQ and $\mathcal{PY}_t$ = NGDPZQ.

Average Imports to GDP Ratio (Nominal): $\frac{C}{Y}$ where,

$$\frac{C}{Y} = \frac{\overline{\mathcal{PC}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PC}_t$ = NCPZQ and $\mathcal{PY}_t$ = NGDPZQ.

Average Nominal Total Tax Revenue to GDP Ratio: $\frac{\mathfrak{T}}{Y}$ where,

$$\frac{\mathfrak{T}}{Y} = \frac{\overline{\mathcal{PT}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PT}_t =$ and $\mathcal{PY}_t =$ NGDPZQ. The monthly tax series are aggregated to create quarterly series and seasonally adjusted using the x12 procedure.

Average Nominal Corporate Tax Revenue to GDP Ratio: $\frac{T_K}{Y} = \tau^k (R - \omega\delta) \frac{K}{Y}$ where,

$$\frac{T_K}{Y} = \tau^k (R - \omega\delta) \frac{K}{Y} = \frac{\mathcal{PTK}_t}{\mathcal{PY}_t},$$

and $\mathcal{PTK}_t =$ FTCPM + FTRWTM + FTTDWTM and $\mathcal{PY}_t =$ NGDPZQ. The monthly tax series are aggregated to create quarterly series and seasonally adjusted using the x12 procedure.

Average Nominal Consumption Tax Revenue to GDP Ratio: $\frac{T_C}{Y} = \tau^c \frac{C}{Y}$ where,

$$\frac{T_C}{Y} = \tau^c \frac{C}{Y} = \frac{\overline{\mathcal{PTC}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PTC}_t =$ FTGSTQ and $\mathcal{PY}_t =$ NGDPZQ. The monthly tax series are aggregated to create quarterly series and seasonally adjusted using the x12 procedure.

Average Nominal Labour Tax Revenue to GDP Ratio: $\frac{T_L}{Y} = \tau^\ell \frac{w\ell}{Y}$ where,

$$\frac{T_L}{Y} = \tau^\ell \frac{w\ell}{Y} = \frac{\overline{\mathcal{PWL}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PWL}_t =$ FTINDVLM and $\mathcal{PY}_t =$ NGDPZQ. The monthly tax series are aggregated to create quarterly series and seasonally adjusted using the x12 procedure.

Average Nominal Total Transfers to GDP Ratio: $\frac{\mathfrak{T}}{Y}$ where,

$$\frac{\mathfrak{T}}{Y} = \frac{\overline{\mathcal{PTR}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PTR}_t$ is collected from the government financial statements and $\mathcal{PY}_t =$ NGDPZQ. The nominal transfers series is seasonally adjusted using the x12 procedure.

Average Net Foreign Debt to GDP Ratio: $\frac{NFA}{Y}$ where,

$$\frac{NFA}{Y} = \frac{\overline{\mathcal{PNFA}_t}}{\mathcal{PY}_t},$$

and $\mathcal{PNFA}_t =$ TIINQ and $\mathcal{PY}_t =$ NGDPZQ.

I. Decomposition

Figure 28 — Baseline Decomposition

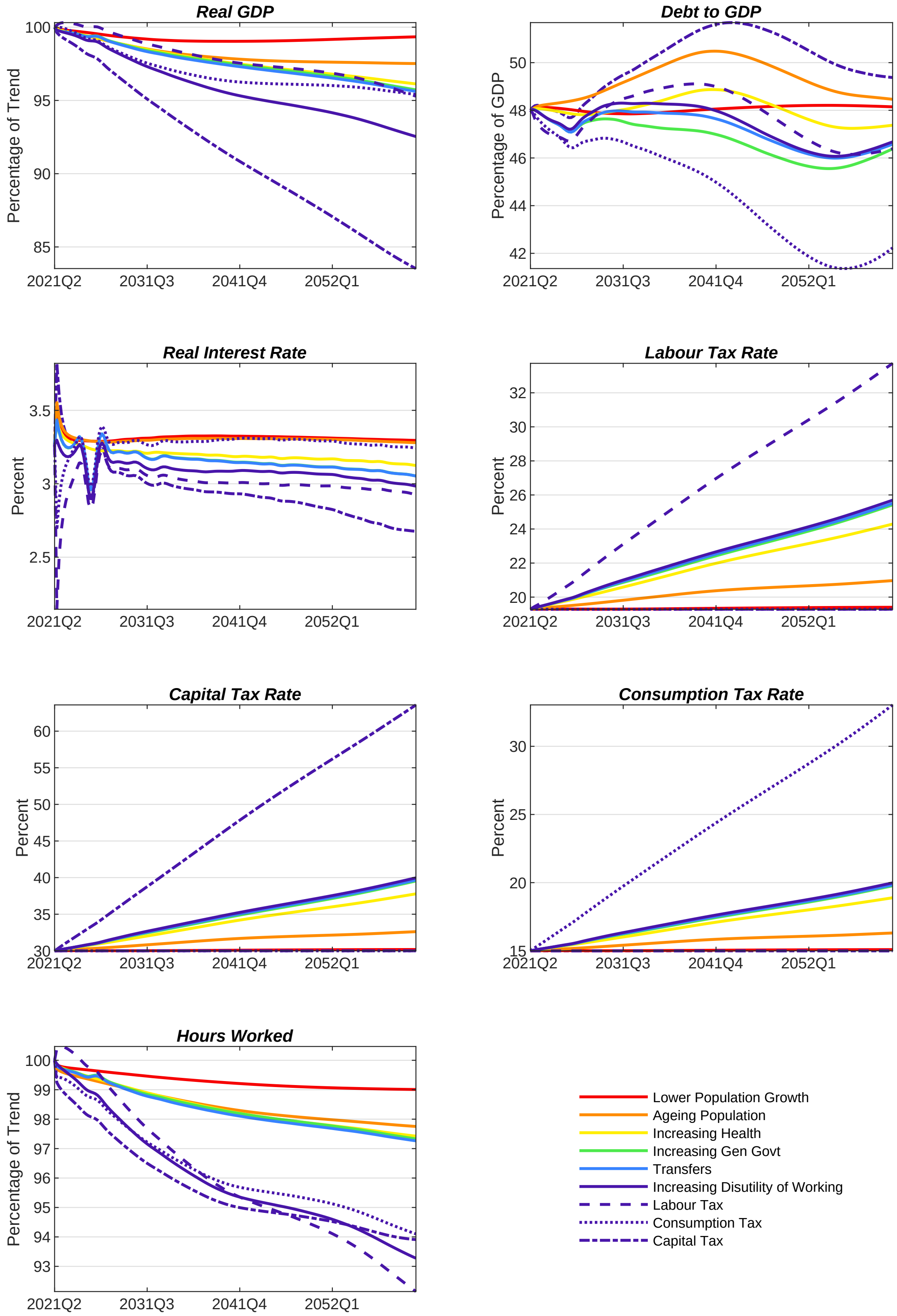


Figure 29 — Baseline Decomposition

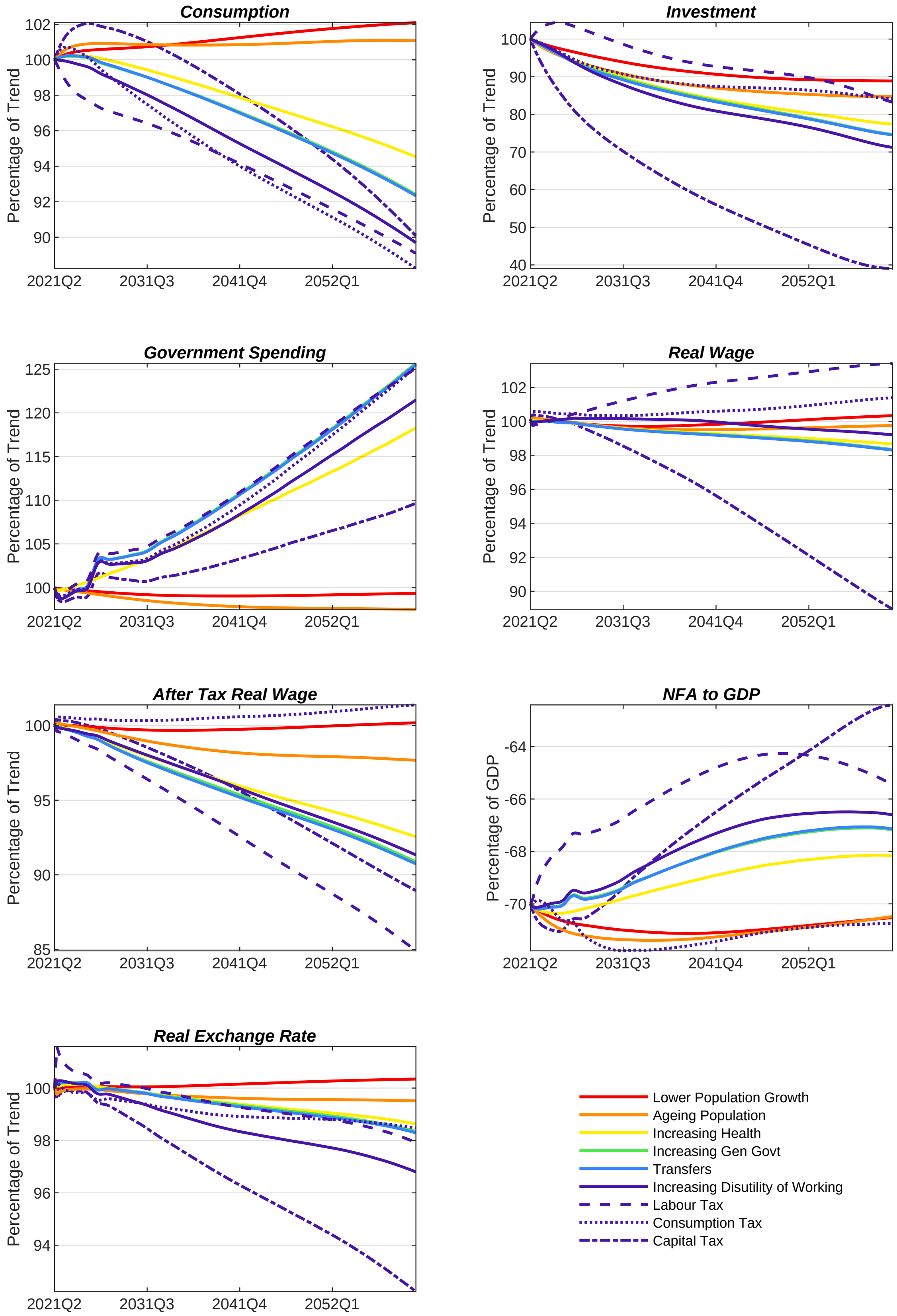
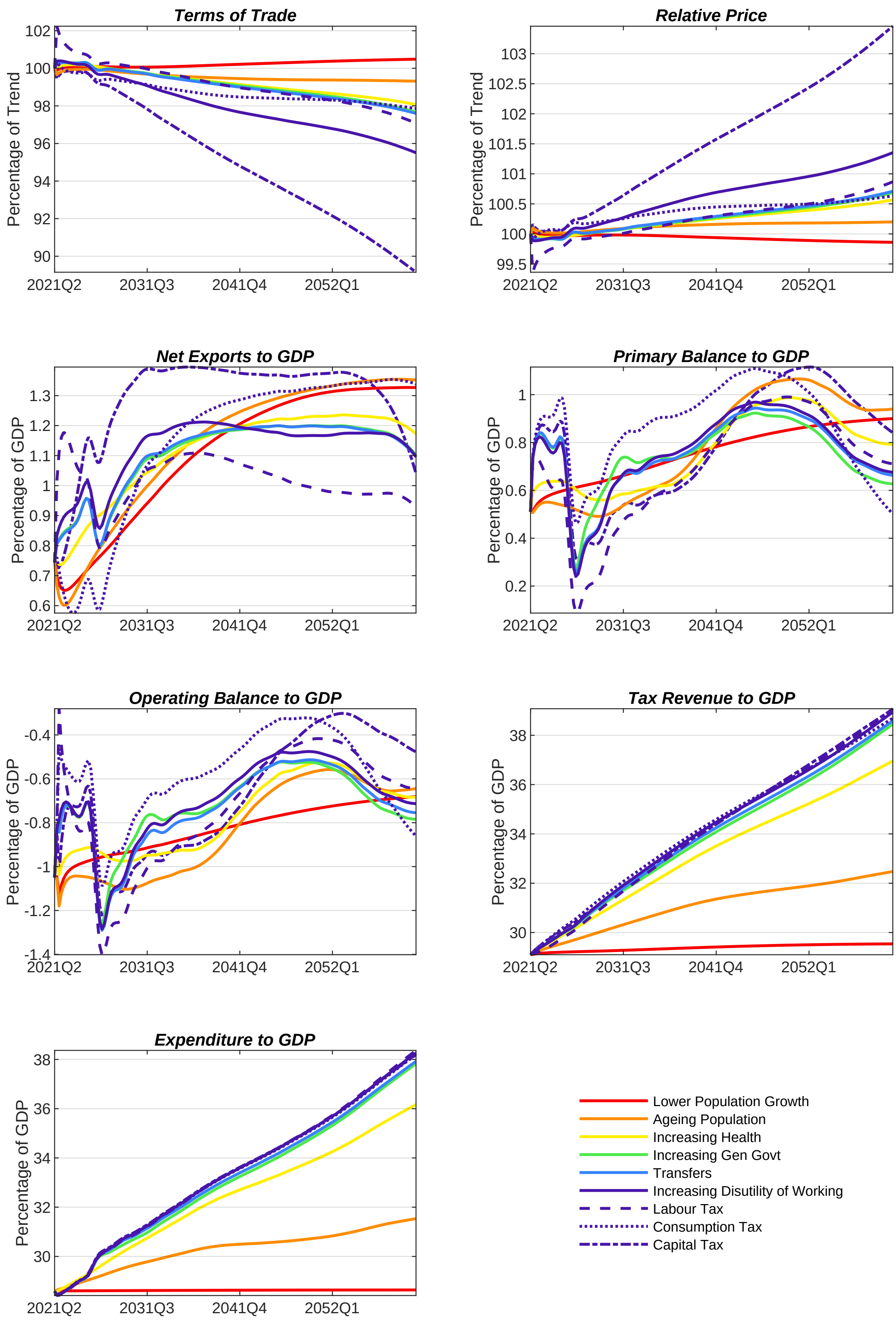


Figure 30 — Baseline Decomposition



Notes: The decomposition graphs show projections where the exogenous variables have been added one by one, so that:

- Lower Population Growth: This is a projection where only the exogenous population growth rate track is added.
- Ageing Population: This is a projection with the lower population growth track and the increasing old-age ratio added, which will capture increasing superannuation expenses.
- Increasing Health: This is a projection with the lower population growth track, the increasing old-age ratio and increasing health expenses added.
- Increasing Gen Govt: This is a projection with the lower population growth track, the increasing old-age ratio, increasing health expenses and increasing general government expenditure added.
- Transfers: This is a projection with the lower population growth track, the increasing old-age ratio, increasing health expenses, increasing general government expenditure and the changes to non-superannuation transfers payments added.
- Increasing Disutility of Working: This is a projection with the lower population growth track, the increasing old-age ratio, increasing health expenses, increasing general government expenditure, the changes to non-superannuation transfers payments and the increasing disutility of working added. This projection is the same as the baseline projection, as it includes all the exogenous tracks.

This means that the difference between any consecutive lines, for the first six lines, will give an order-specific marginal contribution for the addition of that exogenous variable. Different orderings will change the marginal contributions slightly, which is why I average across a couple of different orderings to calculate the approximate contributions of the exogenous variables in Section 5.1.

The last three projections represent counterfactual tax policy scenarios:

- Labour Tax: This projection contains all the exogenous variables in the baseline. However, only labour tax adjusts to pay for the expenditure increases.
- Consumption Tax: This projection contains all the exogenous variables in the baseline. However, only consumption tax adjusts to pay for the expenditure increases.
- Capital Tax: This projection contains all the exogenous variables in the baseline. However, capital tax adjusts to pay for the expenditure increases.

These scenarios show how different tax policies affect the results.

J. Climate Simulations

Figure 31 — A Single Drought

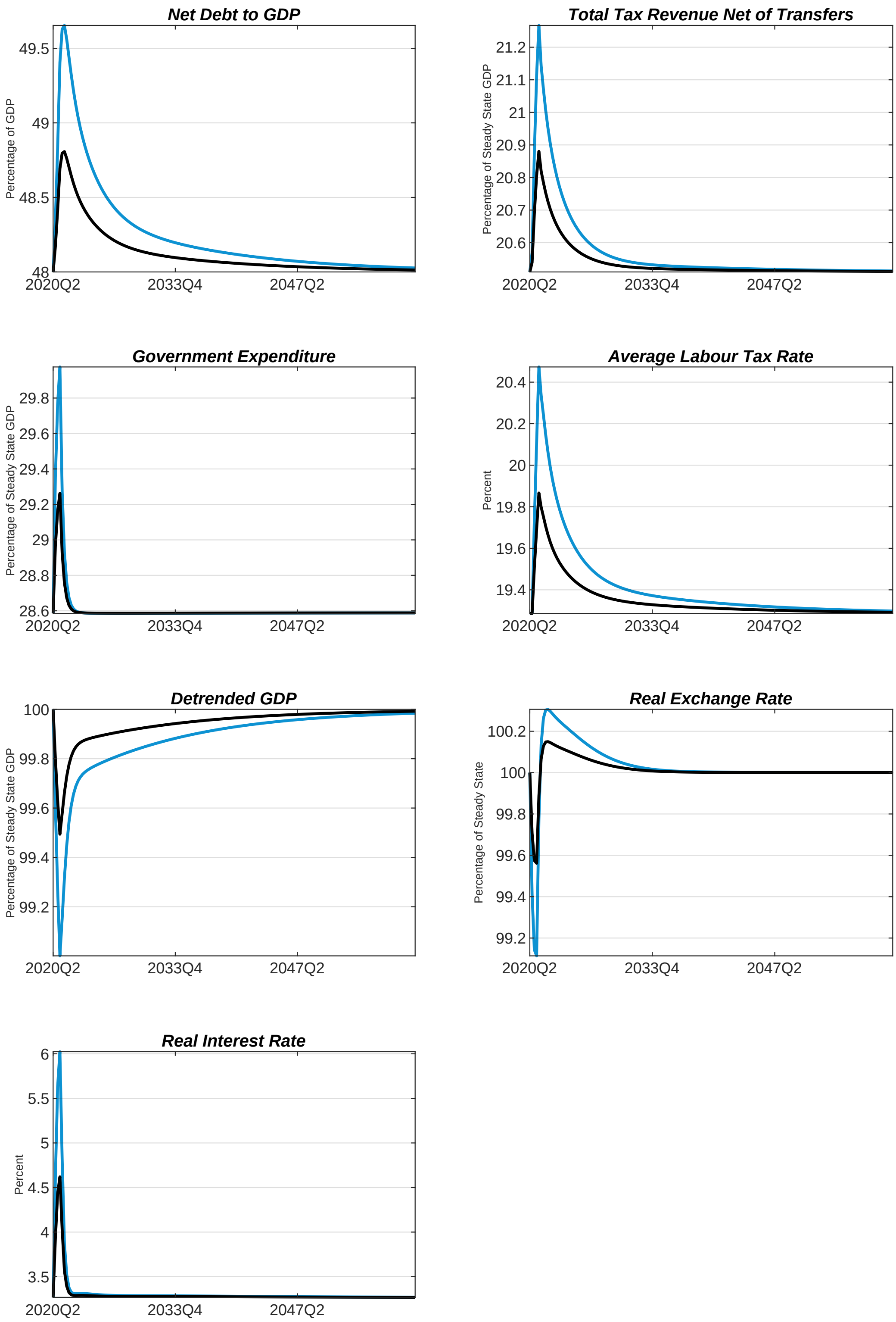


Figure 32 — A Single Storm